Name ______________________________

Ratio and Proportion

Ratios and proportions can be written in fractional form or with colons. The ratio of 4 to 5 can be written as $\frac{4}{5}$ or 4:5. A proportion, two equal ratios, may be stated in words as "4 is to 5 as 8 is to 10" or written $\frac{4}{5} = \frac{8}{10}$ or 4:5 = 8:10	Find the value of n in the proportion: $\frac{4}{6} = \frac{n}{9}$ **cross multiply** $6 \cdot n = 4 \cdot 9$ $6n = 36$ **divide both sides by 6** $\frac{6n}{6} = \frac{36}{6}$ $n = 6$

Write each proportion in fractional form and with colons.

1. 12 is to 8 as 6 is to 4

2. 6 is to 10 as 15 is to 25

3. 70 is to 100 as 14 is to 20

4. 32 is to 12 as 8 is to 3

Find the value of n in each proportion.

5. 6:4 = n:12 ________

6. 1:5 = n:45 ________

7. 2:3 = 6:n ________

8. 5:9 = n:27 ________

9. 5:9 = 40:n ________

10. n:32 = 3:16 ________

11. 8:n = 56:21 ________

12. 48:n = 6:7 ________

13. 2:5 = 24:n ________

14. If a car averages 88 miles in 2 hours, how far will it travel in 3 hours? ________

15. Find the cost of 12 amusement ride tickets that sell at a rate of 3 for $1.50. ________

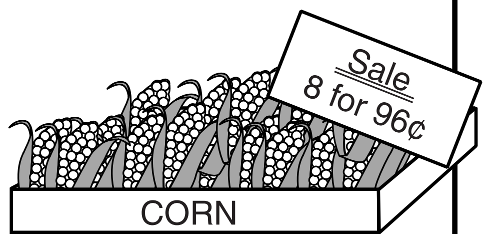
16. The ratio of the width of a rectangle to its length is 4 to 5. If the length is 30, what is the width? ________

17. Oranges sell at a rate of 6 for 72¢. Find the price of 10 oranges. ________

Name ____________________________________

Using Proportion in Word Problems

Corn is selling at 8 for 96¢.
How much will 1 dozen ears of corn cost?



Think: 8 ears is related to 96¢ in the same way that 12 ears is related to how much money?

$$\frac{8}{96} = \frac{12}{n} \rightarrow \frac{8}{96} = \frac{12}{n} \rightarrow \begin{aligned} 8 \times n &= 12 \times 96 \\ 8 \times n &= 1152 \text{ (divide by 8)} \\ n &= 144¢ \end{aligned}$$

1. If 3 tennis balls cost $2.97, how much will 9 tennis balls cost?

2. At the rate of 5 items for 60¢, how many items can you buy for 84¢?

3. To make frosting, Raul needs 2 tablespoons of milk to mix with each cup of powdered sugar. How much milk will he use with 4 cups of powdered sugar?

4. A car travels 14.5 kilometers on 1 liter of gas. How far can it travel on 20 liters of gas?

5. To make concrete, Jon mixes 1 bag of cement with 10 shovelfuls of sand. How much sand needs to be mixed with 20 bags of cement?

6. At the rate of $19.50 per square yard, how much will 12 square yards of carpeting cost?

7. A truck travels 240 miles on 12 gallons of gasoline. At the same rate, how far can the truck travel on a full tank of 22 gallons of gasoline?

8. A chemical formula calls for 6 grams of salt and 3 grams of iron. How many grams of iron would be used with 2 grams of salt?

9. Kia can mow a lawn in 75 minutes. At that rate, how long will it take to mow 3 lawns of the same size?

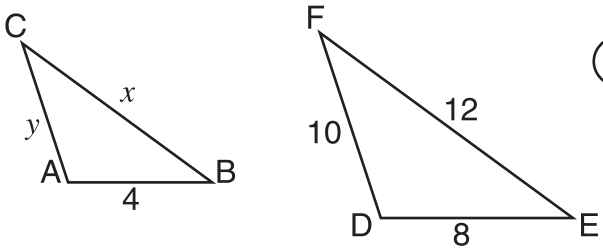
10. If 2 rolls of wallpaper cover 72 square feet, how many rolls will be needed for a room with 360 square feet?

Name ________________________________

Similar Polygons

Two polygons are similar (~)when the corresponding angles are congruent (≅)and the corresponding sides are proportional.

△ABC ~ △DEF

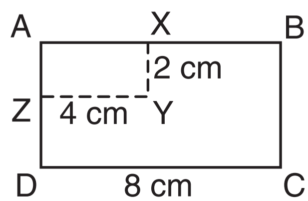


The triangles are similar so you can use proportions to find x and y. Use cross products to find x and y.

$\frac{4}{8} \times \frac{x}{12}$ and $\frac{4}{8} \times \frac{y}{10}$

∠A ≅ ∠D	$\overline{AC} \sim \overline{DF}$	$8 \bullet x = 4 \bullet 12$	$8 \bullet y = 4 \bullet 10$
∠B ≅ ∠E	$\overline{AB} \sim \overline{DE}$	$8x = 48$	$8y = 40$
∠C ≅ ∠F	$\overline{BC} \sim \overline{EF}$	$x =$	$y =$

1. ▭ABCD and ▭AXYZ are similar. How long is $\overline{BC}$?



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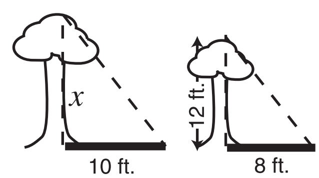
2. If a 12-foot tree casts a 8-foot shadow, how tall is a tree that casts a 10-foot shadow?



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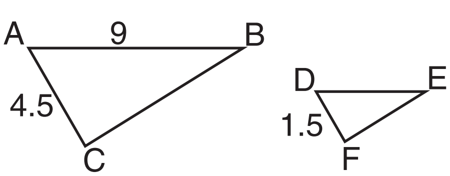
3. △ABC ~ △DEF. Find the length of $\overline{DE}$.



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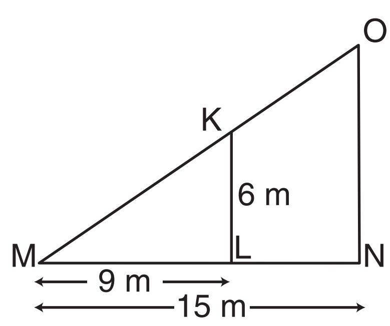
4. △MNO ~ △MLK. Find the length of $\overline{ON}$.



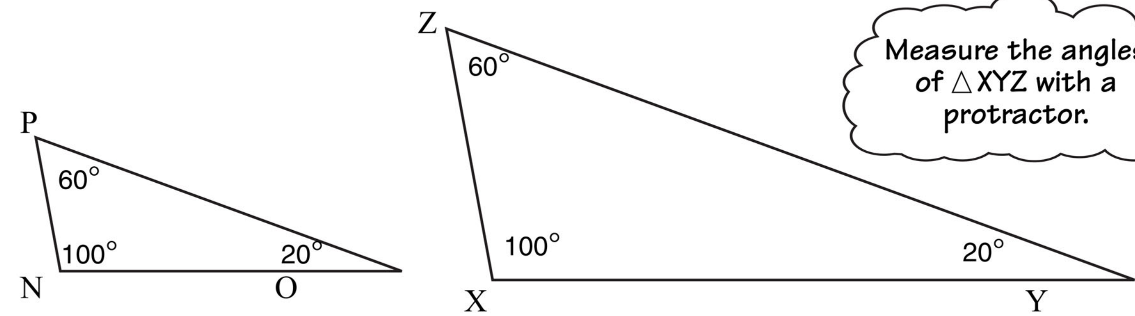
____

Name ______________________________

Corresponding Parts of Similar Figures

Similar figures have the same shape but not the same size. △NOP is similar to △XYZ.

Measure the angles of △XYZ with a protractor.



∠N corresponds to ∠X ∠O corresponds to ∠Y ∠P corresponds to ∠Z

Describe the special relationship between corresponding angles of similar figures:

____

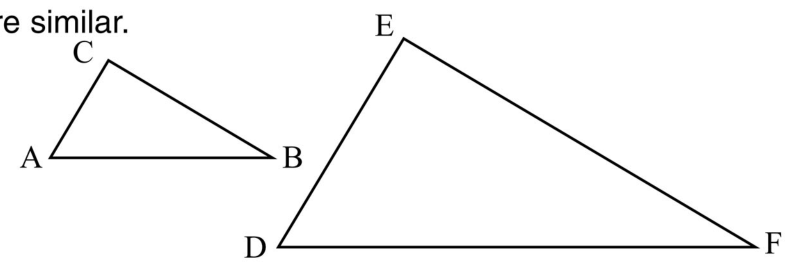
____

1. Triangles ABC and DEF are similar.

∠A = ∠______

∠B = ∠______

∠C = ∠______



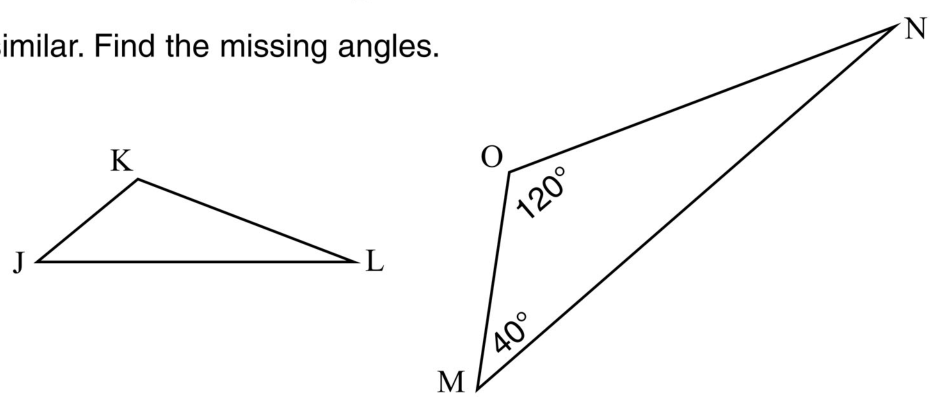
2. The triangles are similar. Find the missing angles.

∠N = ______°

∠J = ______°

∠K = ______°

∠L = ______°



3. Congruent figures have ________
- **A** the same shape but not the same size.
- **B** the same size but not the same shape.
- **C** the same size and shape.

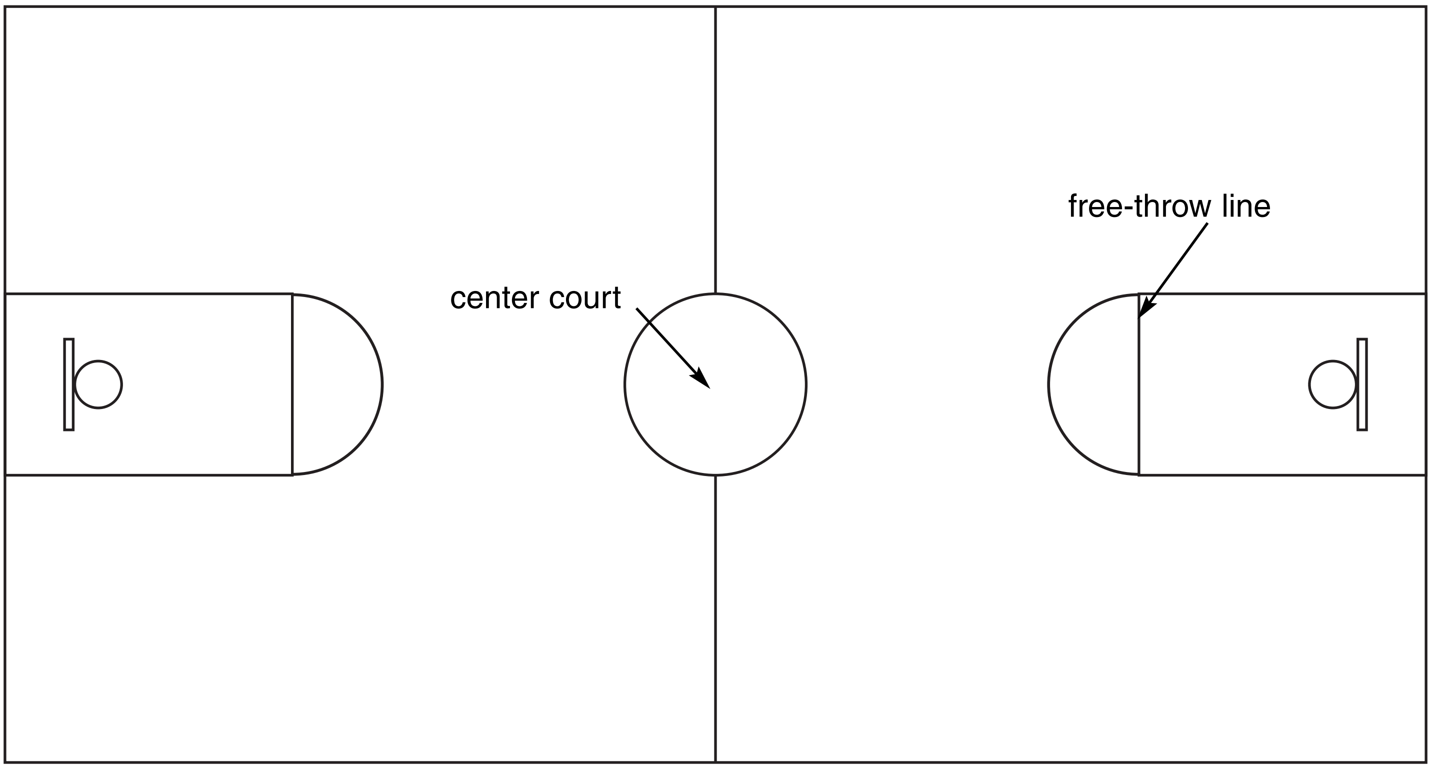
4. Similar figures have ________
- **A** the same shape but not the same size.
- **B** the same size but not the same shape.
- **C** the same size and shape.

Name ______________________________

Scale Drawings

A scale drawing allows you to represent large distances on paper.

Basketball Court



Scale: 1 cm = 5 ft.

Use a centimeter ruler to help answer the questions.

1. length of drawing = ______ cm

actual length of court = ______ ft.

2. width of drawing = ______ cm

actual width of court = ______ ft.

3. width of free-throw line = ______ cm

actual width = ______ ft.

4. distance from center court to free-throw line = ______ cm

actual width = ______ ft.

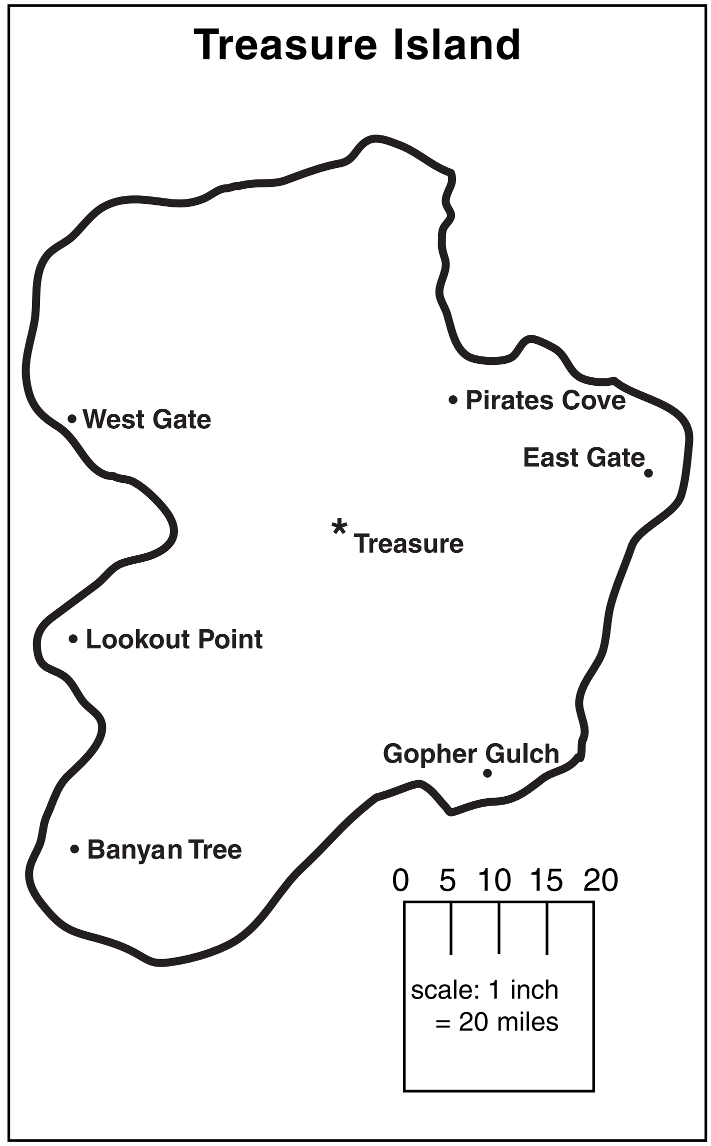
Use a scale of $\frac{1}{4}$ in. = 1 ft. and make a scale drawing of a room of your choice (bedroom, classroom, etc.)

Name ___________________________________

Scale Drawing

A **scale drawing** is an enlarged or reduced drawing of the actual object. The drawing is proportional to the original object. A map is a scale drawing. A scale drawing allows you to represent large distances on a small sheet of paper. The map of Treasure Island is drawn on a scale of 1 inch to 20 miles.

Use a ruler to answer the questions.



1. The distance between the East Gate and the West Gate on the map is 3 inches. What is the actual distance?

____

2. Find the actual distance from the treasure to these landmarks.

a. Gopher Gulch __________

b. East Gate __________

c. West Gate __________

d. Lookout Point __________

e. Pirates Cove __________

f. Banyan Tree __________

3. Find the actual distance from the Banyan Tree to Pirates Cove.

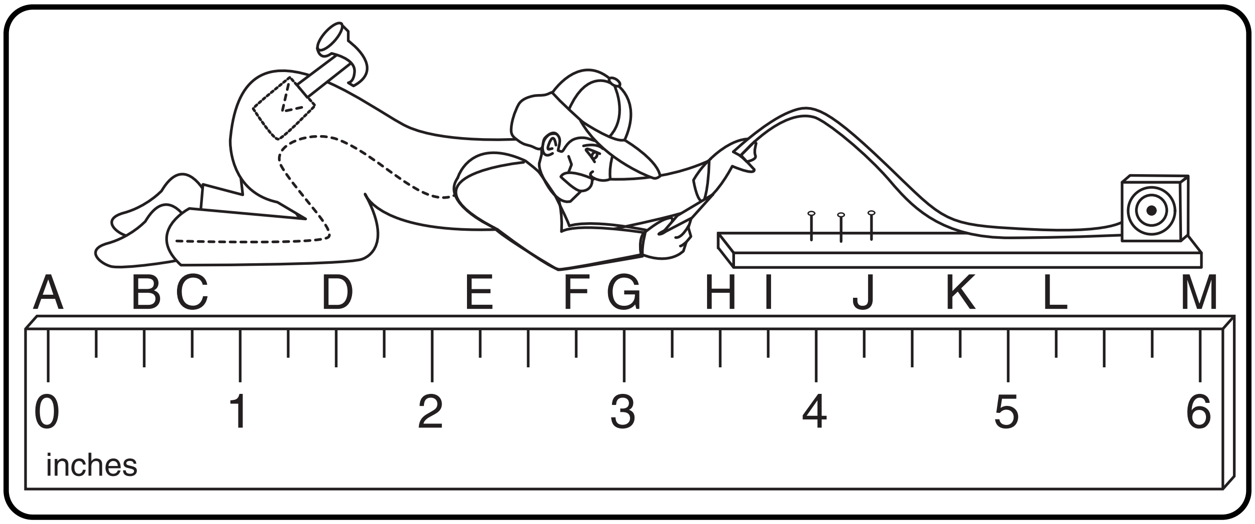
____

4. Find the actual distance from the East Gate to Lookout Point.

____

Name ________________________________

Measuring to the Nearest $\frac{1}{4}$ Inch



1. This ruler is divided into inches, $\frac{1}{2}$ inches and $\frac{1}{4}$ inches. Write the numbers that correspond to each point.

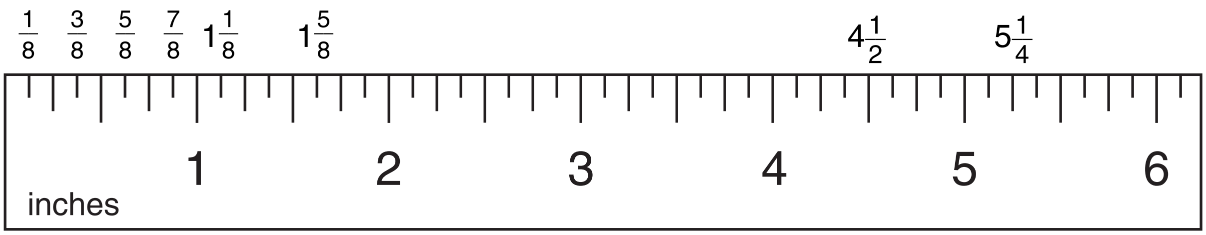
2. Measurements can be written using numbers or words. Fill in the words for each point on the ruler.

	Number	Words
A	0	zero
B	$\frac{1}{2}$	one-half
C		
D		
E		
F		
G		
H		
I		
J		
K		
L		
M		

Name ____________________________________

Measuring to the Nearest $\frac{1}{8}$ Inch

Here is part of a ruler. The whole inch is divided into parts by lines. Each part is $\frac{1}{8}$ of an inch.
There are two abbreviations for inch: in. and ".

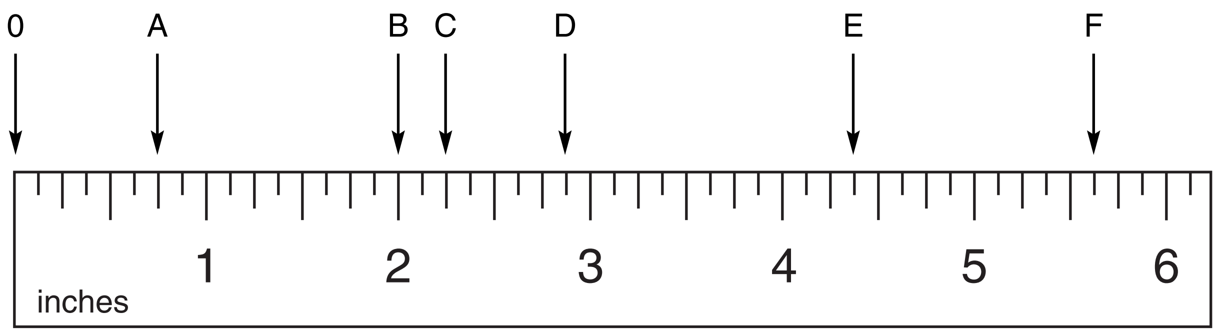


Look at the ruler above to find another name for each of the measurements.

1. $\frac{2}{8}$ in. = _____ in. **2.** $\frac{4}{8}$ in. = _____ in. **3.** $\frac{6}{8}$ in. = _____ in.

4. $\frac{8}{8}$ in. = _____ in. **5.** $5\frac{1}{4}$ in. = _____ in. **6.** $4\frac{1}{2}$ in. = _____ in.

What is the measurement from 0 to each letter?



7. to A __________ in. **8.** to B __________ in. **9.** to C __________ in.

10. to D __________ in. **11.** to E __________ in. **12.** to F __________ in.

13. How far is it from A to B? __________

14. How far is it from A to C? __________

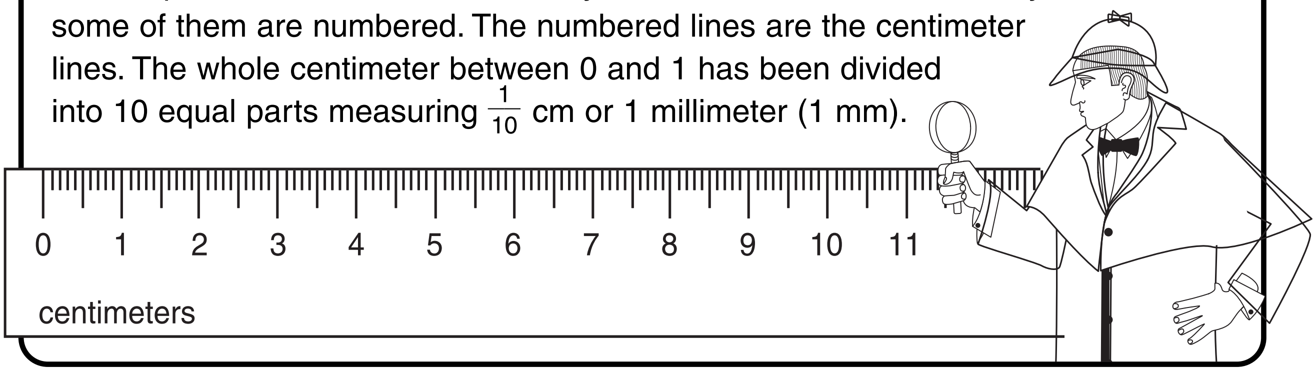
15. How far is it from D to F? __________

16. How far is it from B to F? __________

Name ___________________________________

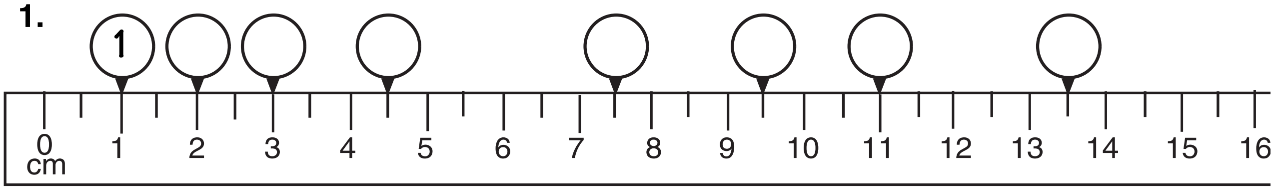
Measuring to the Nearest Millimeter

Here is part of a ruler. There are many little lines on the ruler, but only some of them are numbered. The numbered lines are the centimeter lines. The whole centimeter between 0 and 1 has been divided into 10 equal parts measuring $\frac{1}{10}$ cm or 1 millimeter (1 mm).

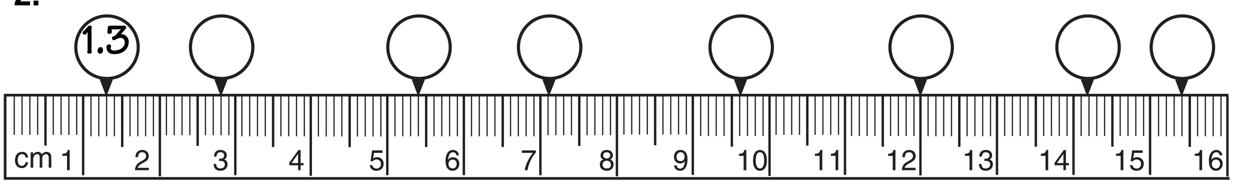


These rulers have been divided into centimeters and parts of a centimeter. What numbers go in the circles?

1.

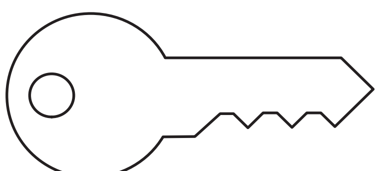


2.



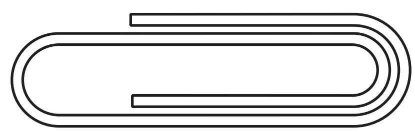
Use a centimeter ruler to measure each object to the nearest centimeter.

3.



________ cm

4.



________ cm

How long is each line to the nearest centimeter?

5. ____

________ cm

6. ____________________________

________ cm

How long is each line to the nearest millimeter?

7. __________________________

________ mm

8. ____________________

________ mm

Name ______________________________

Functions

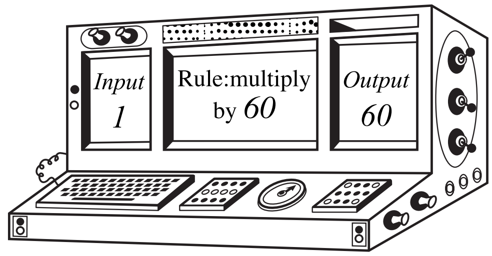
A **function** is a set of ordered pairs that relate to each other by some rule. For each input number, there is only one output number.

Your heart beats 60 times each minute. At that rate, how many times will it beat in 2 minutes? 3 minutes?

The number of heartbeats equals 60 times the number of minutes.

min.	1	2	3
beats	60	120	180

The function machine gives the rule that relates the number of heartbeats to the number of minutes.



Use the function rule to complete each set of ordered pairs.

1. Subtract 3
(8, 5) (7, 4) (6, ___) (5, ___)

2. Multiply by 4
(1, 4) (2, ___) (3, ___) (4, ___)

Give the rule for each set of ordered pairs.

3. (3, 6) (4, 7) (5, 8)
Rule: ______________________

4. (12, 9) (11, 8) (10, 7)
Rule: ______________________

5. (50, 5) (40, 4) (30, 3)
Rule: ______________________

6. (2, 8) (6, 24) (10, ___) (20, ___)
Rule: ______________________

Complete each table.

7. Multiply by 3.

Input (x)	Output (y)
0	
1	
2	
3	
4	

8. Subtract 2.

Input (x)	Output (y)
3	
4	
5	
6	
7	

9. Add 4.

Input (x)	Output (y)
0	
1	
2	
3	
4	

MA-39

Name ____________________________________

Graph of Functions

A **function** is a set of ordered pairs which relate to each other by some rule. Functions may be represented by words, rules, tables or graphs.

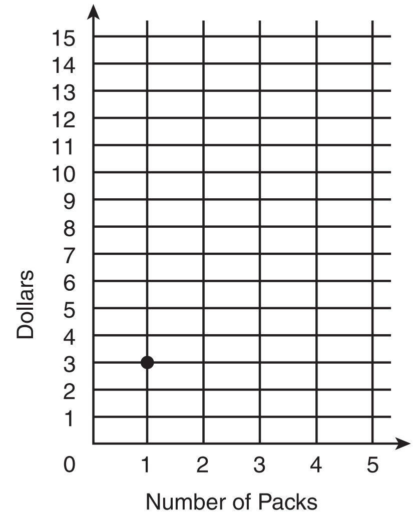
Sharifa collects basketball cards. There was a big sale at Driscoll's Drug Store.

What rule or pattern connects the number of packs to the total dollars needed?

Rule: Multiply the number of packs by 3 to find the total dollars needed.

Show the function on a coordinate grid to make a graph.

COST OF CARDS PER PACK



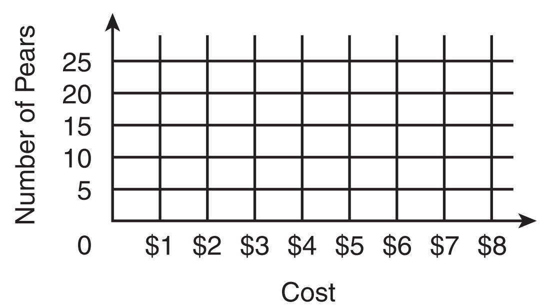
The first point is over 1, up 3. The second point is over ____, up ____. Complete the graph.

Make a table and graph to show the solutions.

1. Pears are on sale at the price of 5 for $2.00. Find the number of pears you can buy for $2.00, $4.00, $6.00 and $8.00.

Cost	$2	$4	$6	$8
No. of Pears	5			

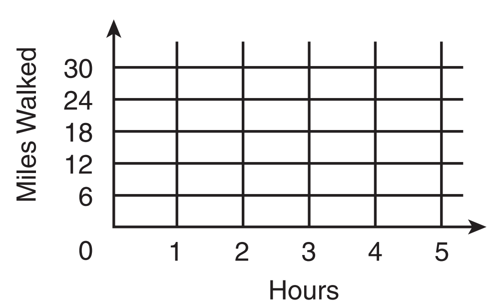
COST OF PEARS



2. You walk at the rate of 6 miles per hour. Show how far you can walk in 1, 2, 3, 4 and 5 hours.

Hours	1	2	3	4	5
Miles	6				

DISTANCE WALKED



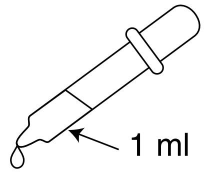
Name ______________________________

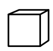
Units of Capacity in the Metric System

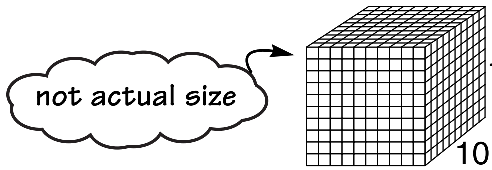
We can also measure capacity in milliliters (ml) and liters (l). Small amounts of liquid are measured in milliliters. Larger amounts are measured in liters.

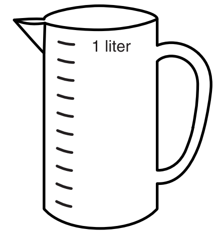
A cube measuring 1 centimeter on each edge will hold 1 milliliter of water.

A cube measuring 10 centimeters on each edge will hold 1 liter of water.





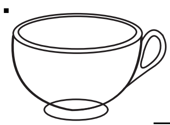




1 liter (l) = 1000 milliliters (ml)

1 liter

Does each container hold more or less than a liter?

1.  __________

2. 

____

3. 

____

Would you measure how much each holds in milliliters or liters?

4. an eyedropper __________

5. a juice pitcher __________

6. the juice of a lemon __________

Complete.

7. 2 l = _________ ml

8. 3000 ml = _______ l

9. 5 l = __________ ml

10. 6000 ml = _______ l

11. 4500 ml = _______ l

12. 2.5 l = ________ ml

13. 1 cm cube = ____ ml

14. 1 l = ____ cm cube(s)

15. 1 ml = ___ cm cube(s)

16. Tom bought 3 cans of orange juice. Each can held 250 ml. Did he buy more or less than one liter of orange juice? _________

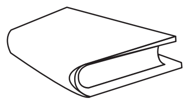
17. Mary bought 60.8 l of gas and 5.4 l of oil. How many liters did she buy in all? _________

Name ______________________________

Metric Units of Weight

The **gram** and **kilogram** are used to measure weight in the metric system.

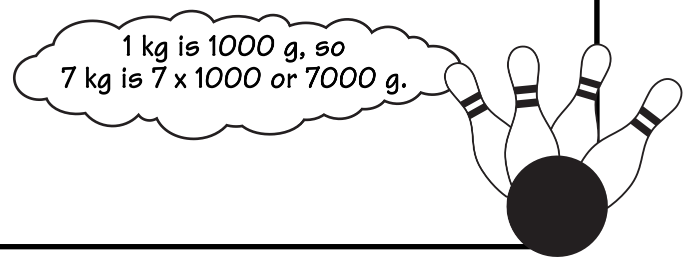
One paper clip weighs about 1 gram.



One textbook weighs about 1 kilogram.

1000 grams (g) = 1 kilogram (kg)

A bowling ball weighs 7 kg. How many grams does the bowling ball weigh?



Would you measure in grams or kilograms?

1. tennis ball ________

2. a television set ________

3. a thumbtack ________

4. a student in your class ________

5. a cat ________

6. a dime ________

Complete.

7. 2 kg = _____ g

8. 3000 g = _____ kg

9. 4 kg = _____ g

10. $3\frac{1}{2}$ kg = _____ g

11. 2500 g = _____ kg

12. 1300 g = _____ kg

13. 500 g = _____ kg

14. $1\frac{1}{4}$ kg = _____ g

15. 200 g = _____ kg

16. 2.3 kg = _____ g

17. 1750 g = _____ kg

18. 3.62 kg = _____ g

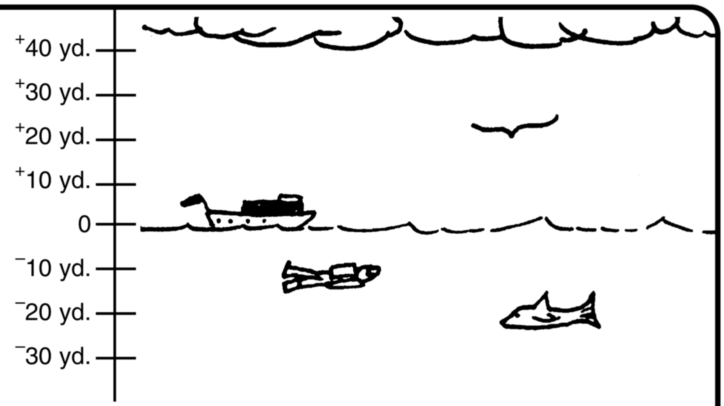
There are 1000 meters in a kilometer and 1000 grams in a kilogram.
What does the word “kilo” mean?
How many liters do you think there are in a kiloliter?

Name ____________________________________

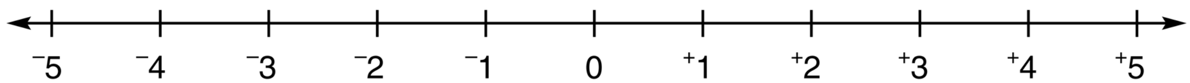
The Set of Integers

The boat is located at sea level (0). The bird is located 20 yards above sea level ($^{+}20$). The scuba diver is swimming 10 yards below sea level ($^{-}10$).

What number locates the cloud cover? The fish?



Whole numbers with positive signs are called positive integers and those with negative signs are called negative integers. The set of integers: {…, $^{-}3$, $^{-}2$, $^{-}1$, 0, $^{+}1$, $^{+}2$, $^{+}3$, …}



1. If $^{-}5$ stands for 5 kilometers south, what does $^{+}10$ stand for? ____________

2. If $^{+}7$ stands for gaining 7 yards in football, what does $^{-}3$ stand for? ________

3. If $^{-}15$ stands for 15 yards west, what number stands for 12 yards east? ______

4. If $^{-}2$ stands for going down 2 floors in an elevator, what number stands for going up 6 floors? ____________________

Write the integer that is:

5. three spaces to the right of 0 ________

6. four spaces to the left of 0 ________

7. seven spaces to the left of 0 ________

8. 13 spaces to the right of 0 ________

9. five spaces to the right of $^{+}2$ ________

10. two spaces to the left of $^{-}1$ ________

11. six spaces to the left of $^{-}3$ ________

12. four spaces to the right of $^{+}3$ ________

13. two spaces to the left of $^{+}1$ ________

14. three spaces to the right of $^{-}2$ ________

15. ten spaces to the right of $^{-}5$ ________

16. four spaces to the left of $^{+}3$ ________

Write how you get to 0 if you start at:

17. $^{+}4$ ____________________

18. $^{-}2$ ____________________

19. $^{-}7$ ____________________

20. $^{+}13$ ____________________

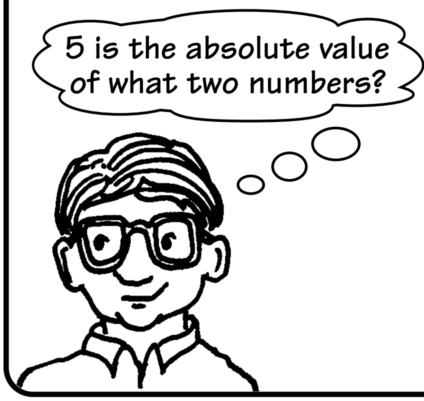
Name ________________________________

Absolute Value

The **absolute value** of a number is the **distance** the number is from zero. Because distance cannot be negative, the absolute value is always positive. The sign for absolute value is | |.

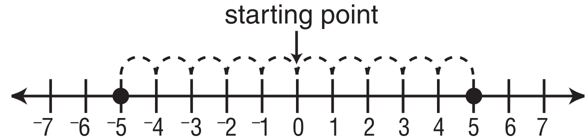
$|n| = 5$

This is read, "The absolute value of n is 5."



We can use a number line to help by thinking, "What two numbers are 5 units from 0?"

Start at zero and move five units in both directions.



We can see that 5 is the absolute value of both $^{-}5$ and $^{+}5$.

Write the absolute value of each number.

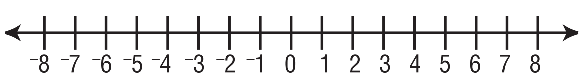
1. $|^{-}17| =$ _____ **2.** $|9| =$ _____ **3.** $|25| =$ _____ **4.** $|^{-}32| =$ _____

Which absolute value is greater?

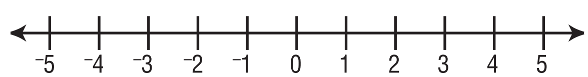
5. $|^{-}8|$ or $|4|$ ________ **6.** $|2|$ or $|^{-}8|$ ________

The absolute value of a number n is indicated. Write the solutions for n and plot the solutions on a number line.

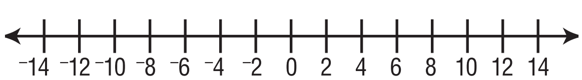
7. $|n| = 6$ $n =$ _____



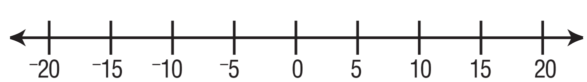
8. $|n| = 4$ $n =$ _____



9. $|n| = 12$ $n =$ _____



10. $|n| = 15$ $n =$ _____



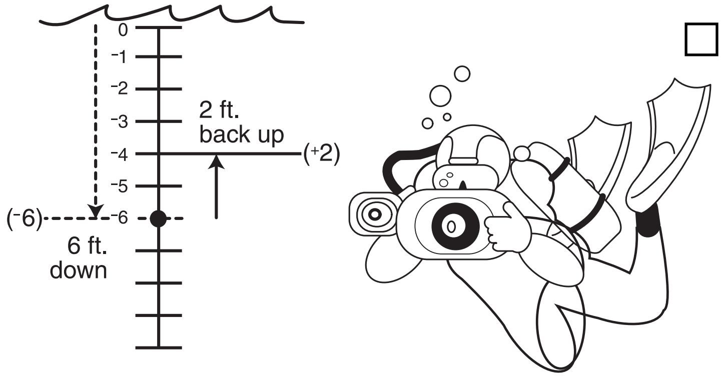
Name ____________________________________

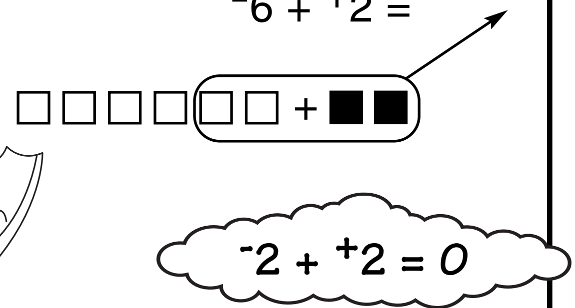
Adding Positive and Negative Integers on a Number Line

You can use models or a number line to add a positive integer and a negative integer.

A scuba diver descended 6 ft. below the water and then ascended 2 ft. Where is the diver?

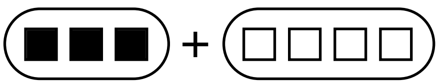
$^{-}6 + {}^{+}2 =$



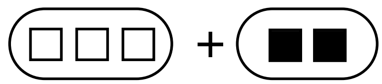


$^{-}6 + {}^{+}2 = {}^{-}4$

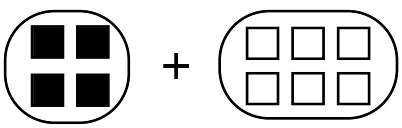
Write the integers. Count to find the sum.

1. 

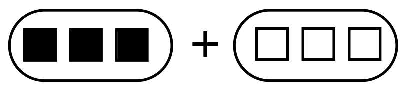
______ + ______ = ______

2. 

______ + ______ = ______

3. 

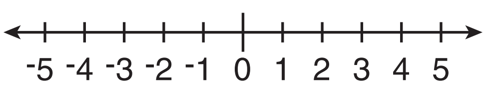
______ + ______ = ______

4. 

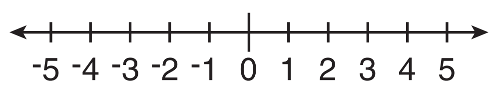
______ + ______ = ______

Show the sum on the number line.

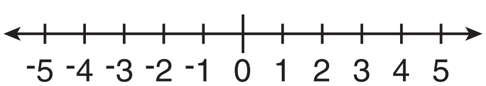
5. $^{-}3 + {}^{+}2 =$ ______



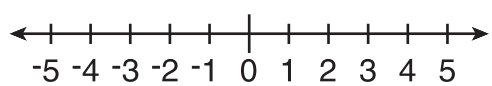
6. $^{+}5 + {}^{-}2 =$ ______



7. $^{+}3 + {}^{-}5 =$ ______



8. $^{+}4 + {}^{-}5 =$ ______



Solve.

9. A football team gained 3 yards on the first down and lost 5 yards on the second down. What was their net loss or gain after two downs?

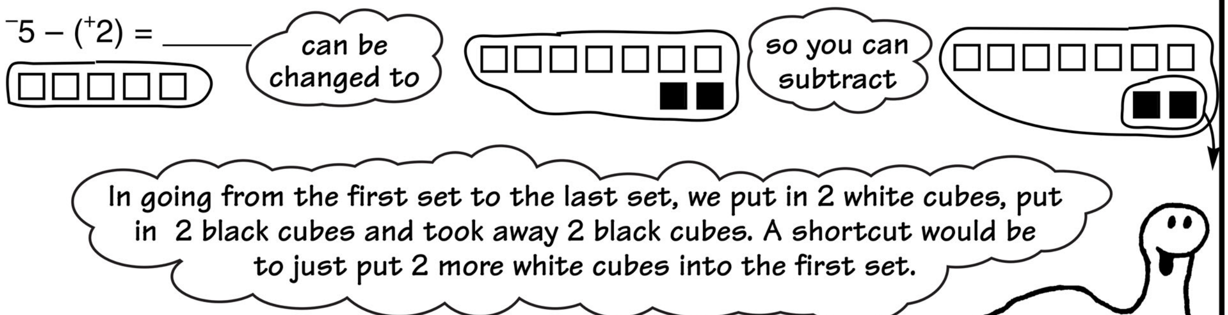
_____ yd.

10. A football team lost 5 yards on the first down, lost 3 yards on the second down and gained 8 yards on the third down. What was their net loss or gain after three downs?

_____ yd.

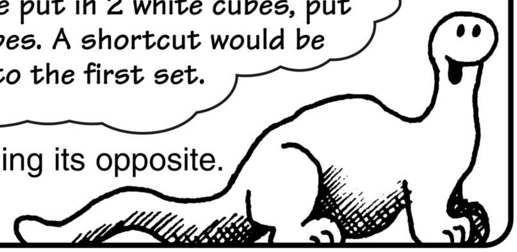
Name ______________________________

Subtracting Integers: A Shortcut



Pattern: Subtracting an integer is the same as adding its opposite.

$^{-}5 - (^{+}2)$ is the same as $^{-}5 + (^{-}2) = {}^{-}7$



Rewrite each subtraction problem as an addition of its opposite.
The first one has been done for you.

1. $3 - (^{-}6) =$

$3 + (^{+}6) = 9$

2. $^{-}5 - (^{-}2) =$ ______________

3. $^{+}4 - (^{+}3) =$ ______________

4. $^{+}4 - (^{-}3) =$ ______________

5. $^{-}3 - (^{-}7) =$ ______________

6. $^{-}7 - (^{+}4) =$ ______________

7. $^{+}2 - (^{+}6) =$ ______________

8. $^{+}5 - (^{-}9) =$ ______________

9. $^{-}6 - (^{-}3) =$ ______________

10. $^{-}8 - (^{+}9) =$ ______________

11. $^{+}6 - (^{+}1) =$ ______________

12. $^{-}9 - (^{-}3) =$ ______________

13. $^{-}40 - (^{+}15) =$ ______________

14. $^{+}61 - (^{-}32) =$ ______________

15. $^{+}84 - (^{+}13) =$ ______________

16. $^{-}50 - (^{-}25) =$ ______________

17. $^{-}13 - (^{+}27) =$ ______________

18. $^{-}12 - (^{-}35) =$ ______________

Name ________________________________

Word Problems

Write a number phrase for each word phrase. Solve.

1. a profit of $20 followed by a loss of $35

   ____

2. a drop of 8° in temperature followed by a drop of 4°

   ____

3. golf scores of 2 under par one day and 6 over par the next day

   ____

4. a ship moving 15 kilometers east then 7 kilometers west

   ____

5. an 18-yard loss on one football play followed by a 12-yard gain on the next play

   ____

6. an increase of 1500 feet in a plane's altitude followed by a decrease of 2400 feet

   ____

7. a decrease of 6.2 kilograms followed by a decrease of 5.7 kilograms

   ____

8. an increase of $0.36 in the price of an item followed by a decrease of $0.21 in the price

   ____

9. an excess of 6.3 centimeters of rainfall one month followed by a deficiency of 4.8 centimeters the next month

   ____

10. a decrease in sales of $800 one week followed by an increase of $1300 the next week

    ____

11. successive losses of 5 yards, 12 yards and 6 yards by a football team

    ____

12. losses of $35 followed by gains of $24 and $45

    ____

13. scores of 4 under par, 3 under par and 5 over par on three successive days

    ____

14. ship on a course moving 30 kilometers south, 25 kilometers north and 12 kilometers south

    ____

Name ______________________________

Multiplying Integers Using Patterns of Multiples

The patterns in the products of multiples can help us find the rules for the sign of a product.

Positive (P) × Positive (P)

$2 \times 3 = 6$
$1 \times 3 = 3$
$0 \times 3 = 0$
P × **P** = ____

PATTERN: Does product increase or decrease by 3?

Negative (N) × Positive (P)

$2 \times 3 = 6$
$1 \times 3 = 3$
$0 \times 3 =$ ____
$^{-}1 \times 3 =$ ____
$^{-}2 \times 3 =$ ____

N × **P** = ____

PATTERN: Does product increase or decrease by 3?

Negative (N) × Negative (N)

$2 \times (^{-}3) = {}^{-}6$
$1 \times (^{-}3) = {}^{-}3$
$0 \times (^{-}3) =$ ____
$^{-}1 \times (^{-}3) =$ ____
$^{-}2 \times (^{-}3) =$ ____

N × **N** = ____

PATTERN: Does product increase or decrease by 3?

Solve. Write the patterns for the multiplication of integers.

1.
$3 \times 4 =$ ____
$2 \times 4 =$ ____
$1 \times 4 =$ ____
$0 \times 4 =$ ____

Pattern:
Positive × Positive =
____

2.
$1 \times 4 =$ ____
$0 \times 4 =$ ____
$^{-}1 \times 4 =$ ____
$^{-}2 \times 4 =$ ____

Pattern:
Negative × Positive =
____

3.
$1 \times (^{-}2) =$ ____
$0 \times (^{-}2) =$ ____
$^{-}1 \times (^{-}2) =$ ____
$^{-}2 \times (^{-}2) =$ ____

Pattern:
Negative × Negative =
____

Solve.

4. $4 \times (^{-}7) =$ _____
5. $(^{-}4)\,(^{-}8) =$ _____
6. $^{-}6 \bullet 8 =$ _____

7. $9 \times 9 =$ _____
8. $^{-}6 \bullet 9 =$ _____
9. $(^{-}8)\,(^{-}9) =$ _____

10. $(^{-}7)\,(7) =$ _____
11. $9 \bullet 7 =$ _____
12. $(^{-}8)\,(^{-}7) =$ _____

Name ______________________________________

Dividing Integers

The patterns for dividing integers can be found by relating division to multiplication.

Multiplication	Related division	Division patterns
$+4 \times +2 = +8$	$8 \div 2 = 4$	positive ÷ positive = positive
$-4 \times (-2) = +8$	$8 \div (-2) = -4$	positive ÷ negative = negative
$-4 \times 2 = -8$	$-8 \div 2 = -4$	negative ÷ positive = negative
$4 \times (-2) = -8$	$-8 \div (-2) = 4$	negative ÷ negative = positive

1. The quotient of two numbers with the same sign is always ________________.

2. The quotient of two numbers with different signs is always ________________.

3. $-48 \div (-8) =$ _____ **4.** $-36 \div 6 =$ _____ **5.** $42 \div (-6) =$ _____ **6.** $63 \div 7 =$ _____

7. $0 \div (-2) =$ _____ **8.** $-40 \div (-8) =$ _____ **9.** $-56 \div 7 =$ _____ **10.** $15 \div (-5) =$ _____

11. $54 \div 6 =$ _____ **12.** $27 \div (-9) =$ _____ **13.** $-36 \div 4 =$ _____ **14.** $12 \div (-1) =$ _____

15. $75 \div (-5) =$ _____ **16.** $-240 \div 3 =$ _____ **17.** $-50 \div (-10) =$ _____ **18.** $-100 \div 25 =$ _____

19. $-46 \div 23 =$ _____ **20.** $144 \div (-12) =$ _____ **21.** $-60 \div 12 =$ _____ **22.** $-54 \div 27 =$ _____

23. $0 \div 16 =$ _____ **24.** $-246 \div 6 =$ _____ **25.** $34 \div (-34) =$ _____ **26.** $560 \div (-70) =$ _____

27. $-75 \div 1 =$ _____ **28.** $-64 \div 16 =$ _____ **29.** $-121 \div (-11) =$ _____ **30.** $105 \div (-35) =$ _____

Name ______________________________

Signs, Terms, Meanings of Basic Operations

Addition puts things together.

$$\begin{array}{r} 7 \\ +\ 3 \\ \hline 10 \end{array}$$

7 ← addends
3 ← addends
10 ← sum

Subtraction takes things apart.

$$\begin{array}{r} 7 \\ -\ 3 \\ \hline 4 \end{array}$$

7 ← minuend
3 ← subtrahend
4 ← difference

Multiplication puts together groups of equal size.

$$\begin{array}{r} 2 \\ \times\ 3 \\ \hline 6 \end{array}$$

2 ← multiplicand
3 ← multiplier
6 ← product

$2 \times 3 = 6$ can also be written as
$2 \bullet 3$ and $2(3) = 6$

Division takes apart groups of equal size.

3 ← quotient
divisor → 2)6 ← dividend

2)6 can also be written as
$6 \div 2 = 3$ and $\frac{6}{2} = 3$

What sign (+, —, x, ÷) goes in the box?

1. 8 ☐ 6 = 48 **2.** 9 ☐ 4 = 13 **3.** 6 ☐ 4 = 2

4. 17 ☐ 18 = 35 **5.** 55 ☐ 11 = 5 **6.** 19 ☐ 2 = 38

7. 42 ☐ 2 = 21 **8.** 10 = 6 ☐ 4 **9.** 9 = 45 ☐ 5

10. 15 ☐ 2 = 13 **11.** 36 = 12 ☐ 3 **12.** 206 ☐ 3 = 618

13. 102 ☐ 19 = 83 **14.** 519 ☐ 3 = 173 **15.** 173 ☐ 3 = 519

16. 30 ☐ 2 = 15 **17.** 198 ☐ 72 = 126 **18.** 126 ☐ 72 = 198

19. Write 6 times 7 three different ways.

________ ________ ________

20. Which is not the correct way to write 24 divided by 6?

A $\frac{24}{6}$ **C** 6)24

B 24 ÷ 6 **D** 6 ÷ 24

21. The difference between 950 and 848 is ______

22. The sum of 84 and 1228 is ______

23. The product of 10 and 22 is ______

24. The quotient of 98 and 2 is ______

25. The product of 5 and 6 plus 4 is ______

26. The quotient of 80 and 2, less 5 is ______

27. The sum of 10 and the product of 5 and 20 is ______

28. The difference between 198 and 46 is ______

Name ______________________________

Writing Equations from Models or Words

An equation is a mathematical sentence with an equal sign. The equal sign tells you that one side of the equation is equal to the other side if the equation is a true equation.

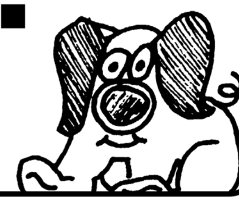
An equation can be written from words.

Kevin added 4¢ to his piggy bank.
He then had 12¢ in his piggy bank.
How much did he have in his bank in the beginning?
piggy bank + 4¢ = 12¢

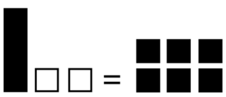
$x + 4 = 12$ $x = 8$

An equation can be written from a model.

$x + 4 = 12$



Write an equation for each model. Use x () as the variable.

1. 

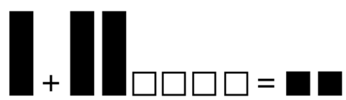
____

2. 

____

3. 

____

4. 

____

5. A number decreased by 10 is 15. ____________

6. Three times a number is 81. ____________

7. Six times the square of a number is 54. ____________

8. 35 decreased by a number is 20. ____________

9. The product of 6 and a number is 72. ____________

10. The quotient of a number and 4 is 12. ____________

11. Twice a number plus 6 is 24. ____________

12. The difference of a number and 5 is 15. ____________

13. Eight less than twice a number is 10. ____________

14. The sum of two times a number and 6 is 24. ____________

15. Two less than seven times a number is 33. ____________

16. The sum of 5 and a number divided by 8 is 12. ____________

17. Two times the sum of 8 and a number is 24. ____________

18. One-half of the product of 6 and a number is 9. ____________

19. Three squared plus a number squared is 25. ____________

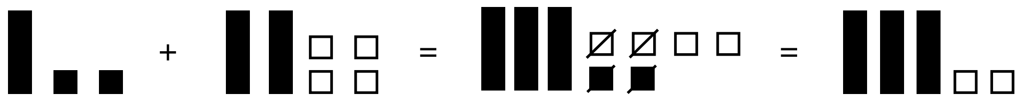
20. The sum of 2 times a number and a number is 45. ____________

What's my number? Can you find the numbers that make problems 1–20 true? You can use trial and error to find each number.

Name ____________________________________

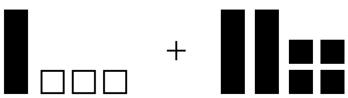
Combining Like Terms Involving Addition

You can use rectangular rods with black and white cubes to combine like terms.



$(x + 2) + (2x - 4) = 1x + 2x + 2 - 4 = 3x - 2$

Draw a picture to combine like terms for problem 1.

1.  =

Finish writing the equation represented by the pictures in problem 1.

2. $(x - 3)$ + (   ) = ____________________

Combine like terms.

3. $(4a - 3) + (a - 4)$

4. $(x + 4) + (2x - 5)$

5. $(2a + 3) + (a - 5)$

6. $(3d + 4) + (2d - 3)$

7. $(3x + 5) - 7$

8. $2x - 6 + 10$

9. $7 - 3x + 5$

10. $(^{-}4n + 2) + (3n + 1)$

Add.

11. $\begin{array}{r} 5x + 7 \\ +\ 3x - 2 \\ \hline \end{array}$

12. $\begin{array}{r} 3c - 5 \\ +\ 5c + 4 \\ \hline \end{array}$

13. $\begin{array}{l} ^{-}7x + 6 \\ +\ 4x \\ \hline \end{array}$

14. $\begin{array}{r} a + 7 \\ +\ 3a - 4 \\ \hline \end{array}$

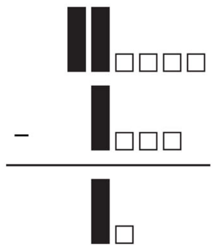
Name ______________________________

Combining Like Terms Involving Subtraction

You can use rectangular rods and black and white cubes to subtract expressions. The difference may also be found by using the pattern for subtracting integers: Add the opposite of each number being subtracted.

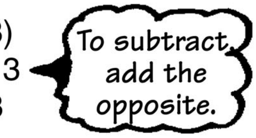
rods and cubes

$$\begin{array}{r} 2x - 4 \\ -\ (x - 3) \\ \hline \end{array}$$



We write:

$2x - 4 - (x - 3)$
$2x - 4 + (^{-}x) + 3$
$2x - x - 4 + 3$
$(2 - 1)x - 1$
$x - 1$



Draw a picture of rods and cubes and write the symbols to subtract.

pictures | symbols

1. $\begin{array}{r} 3x - 5 \\ \underline{-\ (x - 1)} \end{array}$ ____________

2. $\begin{array}{r} 2x - 2 \\ \underline{-\ (x - 1)} \end{array}$ ____________

Subtract and combine like terms.

3. $(4a - 3) - (2a + 5)$

4. $(3x - 5) - (x - 4)$

5. $(4x - 5) - (2x - 3)$

6. $(7d + 9) - (3d + 4)$

7. $(5n - 3) - (n - 2)$

8. $(m - 1) - (2m + 4)$

Subtract. Hint: Add the opposite of the number being subtracted.

9. $\begin{array}{r} 3a - 4 \\ \underline{-\ (2a + 6)} \end{array}$

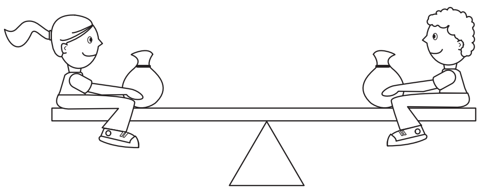
10. $\begin{array}{r} ^{-}7x + 4 \\ \underline{-\ (2x + 3)} \end{array}$

Name ______________________________

Equals Added to or Multiplied by Equals

An equation is like a teeter-totter or a balance scale. Whatever you do to one side of the equation, you must do to the other side to maintain the balance.

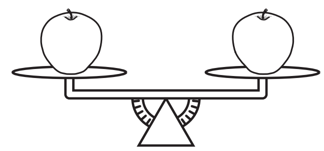
Tracy and Tim weigh the same and balance on a teeter-totter. If Tracy is given an 8 lb. weight to hold, what must Tim be given to maintain their balance?



Tracy = Tim
Tracy + 8 = Tim + ____

Equals added to equals are equal.

When you put 1 apple on one side of a balance scale and 1 apple on the other side of the balance scale, the scale is balanced.



If you double the number of apples on one side, what must you do to the other side to maintain the balance?

Equals multiplied by equals are equal.

The two sides of a scale are balanced. Describe what change must be made to keep the balance after:

1. 15 lb. is added to one side.

____

2. One side is multiplied by 5.

____

3. One side is divided by 3.

____

4. 12 lb. is taken off one side.

____

Indicate what must be done to keep a true equation.

5. $5 + 15 = 5 +$ _____

6. $8 \times 5 = 8 \times$ _____

7. $7 - 4 = 7 -$ _____

8. $12n = 5$
$12n \times \frac{1}{2} = 5 \times$ _____

9. $n \div 6 = 2$
$n \div 6 \times 6 = 2 \times$ _____

10. $18 + x = 23$
$18 + x - 18 = 23 -$ _____

11. $n + 4 = 7$
$n + 4 - 4 = 7 -$ _____

12. $3n = 5$
$3n \div 3 = 5 \div$ _____

13. $\frac{n}{5} = 9$
$\frac{n}{5} \times 5 = 9 \times$ _____

14. $x - 20 = 13$
$x - 20 + 20 = 13 +$ ___

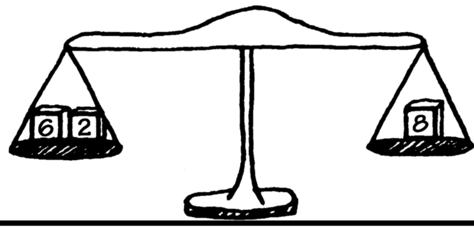
15. $n + 12 = 30$
$n + 12 - 12 = 30 -$ _____

16. $\frac{1}{6}n = 24$
$\frac{1}{6}n \times 6 = 24 \times$ _____

Name ____________________________________

Solving Equations Involving Addition or Subtraction

To solve an equation, think of keeping a scale in balance. When you make any change on one side, you must make the same change on the other side.

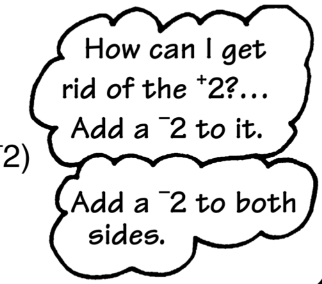


A good strategy is to get the variable alone on one side of the equation.

$n + 2 = 8$

$n + 2 + (^{-}2) = 8 + (^{-}2)$

$n = 6$



Use models of rectangular rods with black and white cubes to solve the equations.

1. ▮□□□ = ■■■■ n = ______ **2.** ▮■■■■ = □□ n = ______

Solve.

3. $n + 5 = 9$ **4.** $n - 7 = 4$ **5.** $4 + n = 6$

6. $n - 5 = 5$ **7.** $n - 7 = 0$ **8.** $n - 15 = 8$

9. $n + 2 = 15$ **10.** $10 + n = 83$ **11.** $n + 15 = 75$

12. $n + 32 = 80$ **13.** $15 + n = 70$ **14.** $n - 8 = 12$

15. $n - 23 = 60$ **16.** $n - 30 = 17$ **17.** $n - 400 = 350$

18. $n - 8 = 7$ **19.** $n + 3 = 22$ **20.** $n - 12 = 24$

21. $n - 3 = 12$ **22.** $48 + n = 68$ **23.** $n - 62 = 30$

24. $n - 4 = 11$ **25.** $8 - n = 6$ **26.** $16 + n = 8$

Name ________________________________

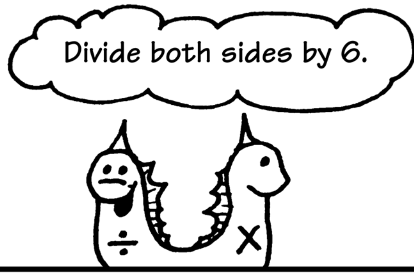
Solving Equations Involving Multiplication or Division

Multiplication and division are **opposite** or **inverse** operations; one undoes the other, e.g., $3 \times 4 = 12$ and $12 \div 3 = 4$. Use the **inverse operation** to solve an equation. Remember to perform the same operation on both sides to maintain a true equation.

$$6n = 24$$

$$\frac{6n}{6} = \frac{24}{6}$$

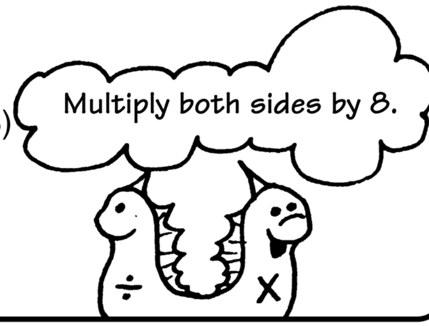
$$n = 4$$



$$\frac{n}{8} = 5$$

$$8\left(\frac{n}{8}\right) = 5(8)$$

$$n = 40$$



Solve each equation by using the inverse operation.

1. $6n = 36$ **2.** $4n = 16$ **3.** $8n = 80$

4. $\frac{n}{5} = 6$ **5.** $\frac{n}{9} = 25$ **6.** $\frac{n}{12} = 4$

7. $\frac{n}{11} = 7$ **8.** $13n = 39$ **9.** $12n = 60$

10. $\frac{n}{15} = 5$ **11.** $12n = 48$ **12.** $\frac{n}{4} = 15$

13. $10n = 50$ **14.** $\frac{n}{8} = 32$ **15.** $2n = 142$

16. $\frac{n}{9} = 9$ **17.** $3n = 45$ **18.** $\frac{n}{77} = 1$

19. $20y = 160$ **20.** $\frac{n}{5} = 80$ **21.** $17n = 51$

Graph the solutions for problems 1–9 on a number line.

Name ______________________________

Addition Property of Equality

If any number is added to both sides of an equation, the new equation is equivalent to the first one.

$$x - 2 = 4$$
$$x - 2 + 2 = 4 + 2$$
$$x = 6$$

Add 2 to both sides of the equation.

Write the number that can be added to the open phrase so that the result is x.

1. $x + 13$ ________ **2.** $25 + x$ ________ **3.** $x + {}^{-}2$ ________

4. $\frac{1}{2} + x$ ________ **5.** $x + {}^{-}6$ ________ **6.** ${}^{-}4 + x + 6$ ________

Write the equivalent sentence resulting when the given number is added to each side of the equation.

7. $x + 6 = 20$; add ${}^{-}6$

$x + 6 + ({}^{-}6) = 20 + ({}^{-}6)$

$x = 14$

8. $x + 3 = 12$; add ${}^{-}3$

____

____

9. $x + {}^{-}20 = 12$; add 20

____

____

10. $12 + x = 3$; add ${}^{-}12$

____

____

11. ${}^{-}4 + x = 2$

____

____

12. $x + 7 = {}^{-}8$

____

____

13. $x + 12 = {}^{-}4$

____

____

14. $x - 8 = 6$

____

____

15. $x + ({}^{-}\frac{3}{4}) = 12$

____

____

16. $x + ({}^{-}\frac{1}{5}) = \frac{3}{5}$

____

____

Name ____________________________________

Multiplication Property of Equality

If both sides of an equation are multiplied by the same number, the new equation is equivalent to the first.

$$2x = 12$$
$$\frac{1}{2} \bullet 2x = \frac{1}{2} \bullet 12$$
$$x = 6$$

You can multiply any number by its reciprocal and the product is 1.

Give the number for the multiplicative inverse of each of the following numbers.

1. 12 ________ **2.** 10 ________ **3.** $\frac{1}{4}$ ________

4. $^{-}6$ ________ **5.** $\frac{2}{3}$ ________ **6.** $^{-}2$ ________

Indicate the number you would multiply each phrase by to get x as the result.

7. $5x$ ________ **8.** $^{-}2x$ ________ **9.** $\frac{1}{3}x$ ________

10. $^{-}\frac{1}{4}x$ ________ **11.** $\frac{3}{5}x$ ________ **12.** $^{-}\frac{2}{5}x$ ________

Write the equivalent sentence resulting when the given number is multiplied on each side of the equation.

13. $3x = 39$; times $\frac{1}{3}$

$\frac{1}{3} \bullet 3x = \frac{1}{3} \bullet 39$

$x = 13$

14. $^{-}5x = 20$; times $^{-}\frac{1}{5}$

____

____

15. $\frac{1}{4}x = {}^{-}6$; times 4

____

____

16. $\frac{2}{3}x = 12$; times $\frac{3}{2}$

____

____

17. $^{-}24 = 4x$; times $\frac{1}{4}$

____

____

18. $^{-}5x = \frac{5}{7}$; times $^{-}\frac{1}{5}$

____

____

19. $^{-}\frac{3}{4}x = \frac{3}{8}$; times $^{-}\frac{4}{3}$

____

____

20. $^{-}24 = 4x$; times $\frac{1}{4}$

____

____

Name ________________________________

Solving Two-Step Equations

Follow these steps to solve equations with more than one operation.

$2n + n - 7 = 23$

1. Combine like terms. ⟶ $3n - 7 = 23$
2. Undo the addition or subtraction. ⟶ $3n - 7 + 7 = 23 + 7$
3. Undo the multiplication or division. ⟶ $\frac{3n}{3} = \frac{30}{3}$

$n = 10$

Solve and check.

1. $2x + x = 15$

$3x = 15$

$x = 5$

Check.

$2 \bullet 5 + 5 = 15$

$15 = 15$

2. $3x + x - 3 = 13$

Check.

3. $5x - 4x + 3 = 22$

Check.

4. $\frac{x}{3} + 1 = 4$

Check.

5. $3x + 5 = 14$

Check.

6. $\frac{x}{2} + 5 = 10$

Check.

7. $4x - 3 = 29$

Check.

8. $\frac{x}{6} - 4 = 1$

Check.

9. $7x + x + 100 = 1700$

Check.

10. $\frac{x}{10} - 1 = 4$

Check.

11. $3x + 5 = 26$

Check.

12. $\frac{x}{4} + 2 = 4$

Check.

13. $2x - 20 = 0$

Check.

14. $\frac{x}{5} + 3 = 7$

Check.

15. $\frac{x}{3} - 4 = 5$

Check.

Name ______________________________

Two-Step Word Problems

Keith's father is 2 years older than his mother. The sum of their ages is 74. How old are Keith's parents?

We are looking for the age of the father and the mother. Start with the age of the mother.

Let x = mother's age

If x = mother's age, how do we represent the father's age?

Let $x + 2$ = father's age

The sum of their ages is 74.

$x + x + 2 = 74$

Combine like terms.

$2x + 2 = 74$

Add ⁻2 to both sides.

$2x + 2 - 2 = 74 - 2$

$2x = 72$

Multiply by the multiplicative inverse.

$\frac{1}{2} \cdot 2x = \frac{1}{2} \cdot 72$

$x = 36$ mother's age

$x + 2 = 38$ father's age

Solve the following problems. Show each step.

1. Jack is 5 years younger than his brother. The sum of their ages is 29. How old are Jack and his brother? Let x = brother's age.

2. Maria is 4 years older than her sister. The sum of their ages is 20. How old are Maria and her sister?

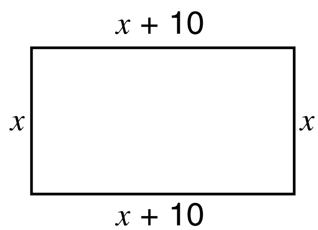
Name ______________________________

Word Problems with Perimeter

The length of a rectangle is 10 feet more than its width. The perimeter is 100 feet. What are the width and the length?

Add the lengths and the widths together to find the perimeter.

Let x = width
Let x + 10 = length



$x + 10 + x + x + 10 + x = 100$

$4x + 20 = 100$

Subtract 20 from both sides.

$4x + 20 - 20 = 100 - 20$

$4x = 80$

Multiply by multiplicative inverse.

$\frac{1}{4} \bullet 4x = \frac{1}{4} \bullet 80$

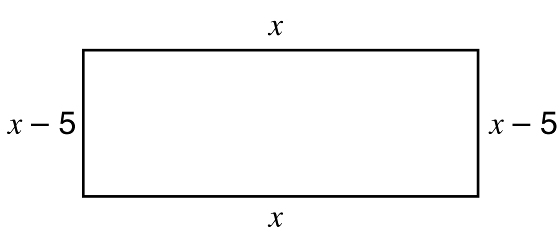
$x = 20$ width

$x + 10 = 30$ length

Solve the following problems. Show each step.

1. The width of a rectangle is 5 centimeters less than the length. The perimeter is 30 centimeters. What are the width and the length?

Let x = length
Let x – 5 = width



2. The length of a rectangle is 2 feet more than its width. The perimeter is 22 feet. What are the width and the length?

Name ______________________________

Applying Principles of Equals

You know that equals added to or multiplied by equals are equals.

Solve:

$2x + 15 = 5x - 6$ — Add $^{-}5x$ to both sides.

$2x + (^{-}5x) + 15 = 5x + (^{-}5x) - 6$

$^{-}3x + 15 = ^{-}6$ — Add $^{-}15$ to both sides.

$^{-}3x + 15 + (^{-}15) = ^{-}6 + (^{-}15)$

$^{-}3x = ^{-}21$ — Multiply both sides by $-\frac{1}{3}$.

$-\frac{1}{3}(^{-}3x) = -\frac{1}{3}(^{-}21)$

$x = 7$

<u>Proof:</u> (put solution for x into equation)

$2x + 15 = 5x - 6$

$2 \bullet 7 + 15 = 5 \bullet 7 - 6$

$14 + 15 = 35 - 6$

$29 = 29$

Use the principle of equals plus equals or equals times equals to solve the following equations. Prove by substituting the value of x back into the original equation. Circle your final answer.

1. $5x + 15 = 39 + 2x$ <u>Proof:</u>

2. $10x - 2 = 8 + 5x$ <u>Proof:</u>

3. $4n + 14 = ^{-}2n + 50$ <u>Proof:</u>

4. $6n - 15 = 84 - 3n$ <u>Proof:</u>

5. $5x - 3x + 18 = 36 - 4x$ <u>Proof:</u>

6. $5n + 10 - n = 12 - n + 3$ <u>Proof:</u>

Name ______________________________

Simplifying Expressions

Expressions can be shortened and simplified by combining like terms.

Simplify: $3(x + 7) + 3x - 7$

Step 1: Remove parentheses: $= 3x + 21 + 3x - 7$

Step 2: Apply commutative property of addition: $= 3x + 3x + 21 - 7$

Step 3: Combine like terms: $= 6x + 14$

Simplify each expression by removing parentheses and combining like terms.

1. $6(x + 2) + 3$

2. $4(x + 2) - 2$

3. $3(x - 6) + 3$

4. $x(2 + 3) - x$

5. $2(^{-}x + 4) + 7$

6. $5(^{-}x - 4) + 2$

7. $2(x + 1) + 3(x + 2)$

8. $4(x - 3) + 2(x + 3)$

9. $3(x - 2) + 5(x - 1)$

10. $7(x + 2) - 2(x + 1)$

11. $6(x + 5) - 8(x - 3)$

12. $9(x + 1) + 4(^{-}2 + x) - 5$

Find the value of each expression above when $x = 3$.

Name ________________________________

Using the Distributive Property To Solve Multi-Step Equations

Solve:

$3(x - 5) - 4(2x - 4) = {}^{-}2x + 22$ — *Use distributive property to remove parentheses.*

$3x - 15 - 8x + 16 = {}^{-}2x + 22$ — *Combine like terms.*

${}^{-}5x + 1 = {}^{-}2x + 22$

${}^{-}5x + 2x + 1 - 1 = {}^{-}2x + 2x + 22 - 1$ — *Add 2x and –1 to both sides of the equation.*

${}^{-}3x = 21$ — *Multiply both sides by $-\frac{1}{3}$.*

$x = {}^{-}7$

Proof: $3({}^{-}7 - 5) - 4({}^{-}14 - 4) = 14 + 22$

${}^{-}36 + 72 = 36$

$36 = 36$

Solve and Prove.

1. $4(3 - 2x) = 7 - (6x + 1)$ Proof:

2. $5(2n + 4) = 2(4n - 1)$ Proof:

Solve.

3. $4(4x - 2) = 2(6x + 2)$

4. $5n - (3n + 2) = 14$

5. $40 = 16 - 4(9 - 3x)$

6. $5n + 2(4n - 5) = 29$

Name ______________________________

Rate Problems

While jogging, Missy checked her pulse and found that her heart was beating at a rate of 160 beats per minute.

Rate: A ratio comparing two different units.

rate = heartbeats per minute

A rate is a ratio whose denominator is always 1.

ratio = $\dfrac{\text{number of heartbeats}}{\text{1 minute}}$

If Missy's heart rate is 160 beats per minute, how many times will her heart beat in 3 minutes?

To find the answer, set up a proportion.

$$\frac{\text{160 beats}}{\text{1 minute}} = \frac{x \text{ beats}}{\text{3 minutes}}$$

$$\frac{160}{1} = \frac{x}{3}$$

Use cross products to solve.

$$\frac{160}{1} \times \frac{x}{3}$$

$$160 \times 3 = 1 \times x$$

$$480 = x$$

Missy's heart will beat 480 times in three minutes.

1. Rick is driving to his grandmother's at a rate of 45 miles per hour. How long will it take him if his Grandmother lives 135 miles away?

____

2. Susan's car can go 28 miles per gallon of gas. If she fills her car with 11 gallons of gas, how far can she drive?

____

3. At the store, hamburger costs $2.89 per pound. How much will 4 pounds of hamburger cost?

____

4. There are 60 minutes in 1 hour. How many minutes are there in 24 hours?

____

5. Karen can read 56 pages in 1 hour. How long will it take her to read a book 616 pages long?

____

6. Richard can drive 26 miles per 1 gallon of gas. If he takes a trip and drives 806 miles, how many gallons of gas will he need to include in his trip budget?

____

7. Bill drove at a rate of 55 miles per hour for 4 hours. How far did he drive?

____

8. A factory can fill 43 bottles in 1 minute. How many bottles can they fill in an hour?

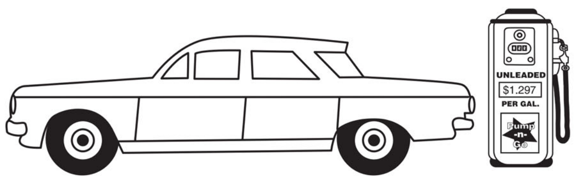
____

Name ____________________________________

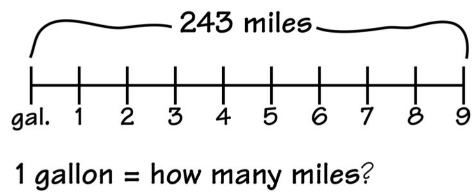
Rates

Gas mileage means the average number of miles a car can travel on one gallon of gasoline. This average number is referred to as “miles per gallon” or m.p.g.

Fred drove 243 miles. He used 9 gallons of gas. How many miles per gallon did he average?



A sketch helps decide on a process.



$$9\overline{)243} = 27$$

18
63
63

27 m.p.g.

Find the m.p.g. Write the letter of the problem above the answer.

What is Dracula’s favorite form of transportation?

___	___	___	___	___		___	___	___	___	___	___
18	30	22	22	12		9	22	18	8	30	32

B. The Sunray that Paula drives went 144 miles on 8 gallons. Find her m.p.g.

____

E. The Brauns drove 672 miles on a vacation. They used 21 gallons of gas. They were gone for 10 days. Find the m.p.g.

____

L. The Firelark advertises it can go 360 miles on one 12-gallon fill-up. What is the m.p.g. of the Firelark?

____

A. Vicki drove 235 miles. Her gas fill-up cost $23.50. What was her cost per mile?

____

D. Charles’ new pickup used 12 gallons of gas to go 144 miles. Find the m.p.g.

____

I. Ralph’s 1986 Road Hog used 15 gallons of gas to go 120 miles. Find his m.p.g.

____

M. Ramona drove her pickup 234 miles and used 26 gallons of gas. What was the m.p.g.?

____

O. The Fishes used 35 gallons of gas on a 770 mile trip. What was the m.p.g.?

____

Name ______________________________

Solving Round-Trip Motion Problems

Some motion problems involve making round trips. They are based on the equation $d = rt$ where d = distance, r = rate of travel and t = time.

Matt and Zak are planning a bike trip. They know they can average 10 miles per hour going and can make the return trip at 8 miles per hour. If they want to be gone a total of 6 hours, how far can they go in one direction before turning back?

1. Underline the question and circle the facts.
2. Set up variables for the unknowns:
   Let t = time outbound and $6 - t$ = time for return trip.
3. Make a distance table.

	Rate	Time	Distance
out	10	t	$10t$
back	8	$6 - t$	$8(6 - t) = 48 - 8t$

4. Set up an equation from the information given:

$$10t = 48 - 8t$$

$$18t = 48$$

$$t = 2\frac{12}{18} = 2\frac{2}{3} \text{ hours outbound}$$

$$6 - t = 6 - 2\frac{2}{3} = 3\frac{1}{3} \text{ hours return trip}$$

The distance out and the distance back must equal each other.

To find the one-way distance:

$$(2\tfrac{2}{3})(10) = (\tfrac{8}{3})(10) = \frac{80}{3} = 23\frac{1}{3} \text{ miles}$$

The question asks for the one-way distance.

Solve these problems following the steps in the example above. Check your answers by returning to the table.

1. A canoe travels downstream at 12 mph and upstream to the same spot at 4 mph. How far did the canoe go downstream during a 7-hour round trip?

__________ miles

2. Kelly averaged 50 mph on the way to a campsite and 40 mph on the way back. The round trip took $13\frac{1}{2}$ hours. How far away is the campsite?

__________ miles

3. A roundtrip on a sailboat took 6 hours. On the trip out the boat sailed at 5 miles per hour and on the way back it sailed twice as fast. How far out did the sailboat go before turning around?

__________ miles

4. Maria drove a trailer to a campsite at a rate of 35 miles per hour and returned at a rate of 40 miles per hour. Find Maria's time going and returning if the time returning was one hour less than the time going.

going = __________ hours

returning = __________ hours

Name ____________________________________

Mixed Practice with Inequalities

Solve. Graph the solutions.

1. $x - 3 > {}^{-}5$

2. $3x + 2 \geq 2x + 4$

3. $2x + 7 < 15$

4. $\frac{x}{4} - 2 \leq {}^{-}1$

5. $6x < 3x + 12$

6. $3x - 9 \geq {}^{-}12$

7. ${}^{-}x > 2$

8. ${}^{-}3x + 4 \leq {}^{-}5$

9. $2x > 3x + 4$

10. $\frac{{}^{-}x}{2} < 2$

11. $\frac{{}^{-}x}{4} \geq \frac{1}{2}$

12. $\frac{x}{6} < \frac{{}^{-}1}{3}$

13. When some number has two added to it, the result is less than or equal to 5.

14. The opposite of some number less 4 is greater than ${}^{-}2$.

15. ${}^{-}3$ times some number is less than 9.

Name ______________________________

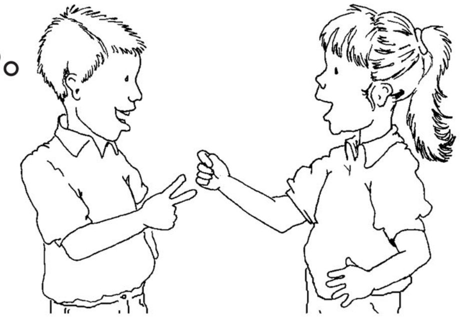
Order of Operations

Mike and Tami found different answers for the problem 4 • 2 + 5. Who was right?

The answer is 28. First I added the 2 + 5, then I multiplied the sum by 4.

$4 \bullet \underbrace{2 + 5} =$

$4 \bullet 7 = 28$



The answer is 13. First I multiplied the 4 • 2. Then I added the 5.

$\underbrace{4 \bullet 2} + 5 =$

$8 + 5 = 13$

The answer to an expression having mixed operations (+, –, ×, ÷) will come out differently depending on which operation you do first. Mathematicians do the operations in this order:

1. **M**ultiply and **D**ivide from left to right. 20 – 16 ÷ 2 = 20 – 8 = 12 *Divide before subtracting.*
2. **A**dd and **S**ubtract from left to right.

This rule is sometimes called the **M**y **D**ear **A**unt **S**ally rule (**MDAS**). 25 + (5)(2) = 25 + 10 = 35 *Multiply before adding.*

Solve. Perform the operations in the correct order.

1. 3 + (6)(4) _____ **2.** 13 + 4 – 2 _____ **3.** 3(4) – 6 _____

4. 8 + (12)(2) _____ **5.** 10 – 12 ÷ 4 _____ **6.** 64 – 14(2) _____

7. 5 + 6 ÷ 3 _____ **8.** 4(6) – 2 • 12 _____ **9.** 6 + 7 • 2 _____

10. 40 – 24 ÷ 4 _____ **11.** 3 + 6 • 5 _____ **12.** 12 ÷ 4 – 2 _____

13. 6 • 4 – 2 • 6 _____ **14.** 10 ÷ 5 + 8 _____ **15.** 5 • 4 ÷ 2 _____

16. 81 ÷ 9 – 5 _____ **17.** 10 ÷ 2 + 5 • 3 _____ **18.** 18 ÷ 9 + 2 • 3 _____

19. 10(6) – 12 ÷ 4 _____ **20.** 14 + 8(4) – 10 _____ **21.** 70 – 15 ÷ 3 _____

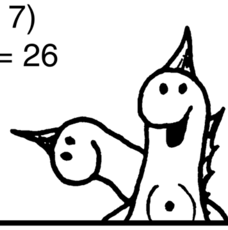
22. 30 – 20 ÷ 5 + 4 _____ **23.** 6 • 8 – 3 • 4 + 2 • 3 _____ **24.** 3(20) – 10(3) + 5 _____

Name ______________________________

Parentheses and Order of Operations

In evaluating an expression, always perform the operation indicated within the parentheses first.

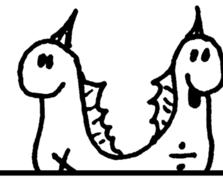
$5 + (3 \bullet 7)$
$5 + 21 = 26$



After all parentheses are renamed, the operations should be performed in this order:

1. multiply or divide from left to right
2. add or subtract from left to right

$36 - 18 \div 2$
$36 - 9 = 27$



Perform the operations in the correct order.

1. $5 \bullet (4 + 6) =$ ______

2. $(10 + 8) \div 2 =$ ______

3. $(7 - 2) \bullet (10 - 5) =$ ______

4. $6 + (3 + 7) =$ ______

5. $5 \div (3 + 7) =$ ______

6. $10 + (12 \div 3) =$ ______

7. $5(3) + 2(8) =$ ______

8. $7(4) - 3(5) =$ ______

9. $4(6 + 3) =$ ______

10. $3(6 - 1) + 4 =$ ______

11. $8 - 2(4) =$ ______

12. $18 + (2 \bullet 4) - (3 \bullet 4) =$ ______

13. $\frac{5(6)}{3} =$ ______

14. $\frac{10}{(3 + 2)} =$ ______

15. $\frac{12}{4} + 8 =$ ______

16. $26 - \frac{6(2)}{3} =$ ______

17. $\frac{3(2 + 5)}{7} =$ ______

18. $\frac{6(4)}{3} + 3(7) =$ ______

19. $(15 + 2) \bullet 10 - 9 \div 3 =$ ______

20. $18 \div 9 + 4 \bullet 7 =$ ______

21. $6 + (24 \div 4) - 12 =$ ______

22. $(13 \bullet 5) \div (10 + 3) =$ ______

23. $7(3 + 5) - 2(3 + 4) =$ ______

24. $24 \bullet 2 + 10 \div 2 - 6 =$ ______

Name ______________________________

Powers and the Order of Operations

When we have powers in an expression, the order of operations becomes:

P and **E**	Remove **P**arentheses and **E**xponents.
M and **D**	**M**ultiply and **D**ivide from left to right.
A and **S**	**A**dd and **S**ubtract from left to right.

Please **E**xcuse **M**y **D**ear **A**unt **S**ally

1 **P and E** — Remove **P**arentheses and **E**xponents: $\frac{(2^3 + 4)}{6} + 8 \div 4 =$

2 **M and D** — **M**ultiply and **D**ivide: $\frac{12}{6} + 8 \div 4 =$

3 **A and S** — **A**dd and **S**ubtract: $2 + 2 = 4$

Evaluate the following expressions. Show your work.

1. $5^2 \times 3 =$

2. $\frac{(3^3 + 3)}{10} =$

3. $7^2 + 4 \times 2 =$

4. $8^2 + 7 - 2 \times 3 =$

5. $(2 + 7) \times 4^2 =$

6. $(1^3 + 2) \times (3 \times 4) =$

7. $2^2 \times 7 - 4 =$

8. $(50 - 6^2) \times 2 =$

9. $10 \times 9 - 3^2 \div 3 =$

10. $10^2 \times 4 \div 2 =$

11. $(6^2 + 3^2) - 5 \times 4 =$

12. $8^2 \times 4 \div 8 =$

13. $(9^2 \div 3) + (4^2 - 12) \times 2 =$

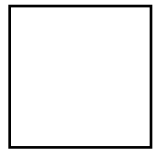
14. $(2^3 + 2) \div 5 - 1 =$

Name ______________________________________

Rational Numbers with Exponents

The area of a square is found by squaring the length of a side.

Example: Find the area of a square having sides of $\frac{3}{4}$ inch.



$\frac{3}{4}$ in.

Method 1

Area = $\left(\frac{3}{4}\right)^2$

$= \frac{3}{4} \times \frac{3}{4}$

$= \frac{9}{16}$ sq. in.

Method 2

Area = $\left(\frac{3}{4}\right)^2$

$= \frac{3^2}{4^2}$

$= \frac{9}{16}$ sq. in.

Simplify by removing exponents.

1. $\left(\frac{4}{7}\right)^3 = \frac{64}{343}$ **2.** $\left(\frac{7}{8}\right)^2 =$ **3.** $\left(\frac{2}{3}\right)^4 =$ **4.** $\left(\frac{1}{2}\right)^5 =$

5. $(0.5)^2 =$ **6.** $(0.4)^3 =$ **7.** $(0.4)^2 =$ **8.** $(1.5)^2 =$

9. Gage is using square tiles to tile his bathroom floor. Each tile is $2\frac{1}{2}$ inches per side. What is the area of each tile?

__________ sq. in.

10. Lydia is buying material to make napkins. If each napkin is to be a square with a side of $\frac{1}{3}$ yard, how many square yards is each napkin?

__________ sq. yd.

Multiply. Leave exponents in your answer.

11. $\left(\frac{2}{7}\right)^4 \times \left(\frac{7}{3}\right)^3 = \frac{2^4}{7^4} \times \frac{7^3}{3^3}$

$= \frac{2^4 \times 7^3}{7^4 \times 3^3}$

$= \frac{2^4}{7^1 \times 3^3}$

12. $\left(\frac{3}{6}\right)^5 \times \left(\frac{6}{3}\right)^4 =$

13. $\left(\frac{7}{9}\right)^2 \times \left(\frac{9}{6}\right)^3 =$

Name ________________________________

Multiplication of Constants and Monomials with Powers

Power of a power: For any number x and integers a and b, $(x^a)^b = x^{ab}$

$(3^2)^4$ means $(3^2)(3^2)(3^2)(3^2) = 3^8$.

$(x^4)^2 = (x^4)(x^4) = x^8$

Power of a product: For any numbers x and y and any integer a, $(xy)^a = x^a y^a$

$(xy)^3$ means $(xy)(xy)(xy) = x^3y^3$

Power of a monomial: For all numbers x and y and integers a,b and c $(x^a y^b)^c = x^{ac}y^{bc}$

$(2m^2n)^3 = (2)^3 (m^2)^3 (n)^3 = 8m^{\square}n^{\square}$

$(x^5y^3)^4 = (\_\_\_\_\_)(\_\_\_\_\_)(\_\_\_\_\_)(\_\_\_\_\_) = x^{\square}y^{\square}$

Simplify.

1. $(7ab)(4ab)$

2. $(-gh^2)(2g^2)$

3. $(9x^2y^2)(8xy^3)$

4. $(^-8c^2d)(^-2cd^3)$

5. $(7mn)(2np)(3mnp)$

6. $(3^2)^4$

7. $(2xy^2)^3$

8. $(m^3)^5$

9. $(2x^2y^3)^2$

10. $(a^5b^3c^2)^3$

11. $(mn)^4$

12. $(^-3k)^3$

13. $(^-3ab)(^-6a)(^-1ab^2)$

14. $(x)(3x^4y)(8xy)$

Name ______________________________

Multiplying Monomials

Multiply monomials in three steps.

Example: $(5x^2)(^-3x^4)$

Step 1: Determine the sign. (+) (–) (–) (positive × negative = negative)

Step 2: Multiply the numbers. $(5 \times {}^-3 = {}^-15)$

Step 3: Multiply the variables. $(x^2 \times x^4 = x^6)$

Write the answer. $^-15x^6$

Find the product.

1. $(4n^2)(^-7n)$ __________

2. $8(^-10m)$ __________

3. $x^4 \bullet x^6$ __________

4. $(4h)(5h^2)$ __________

5. $(6n^4)(^-7n^3)$ __________

6. $(8m^2)(^-4mn)$ __________

7. $(x^2)^2$ __________

8. $(x^2)^3$ __________

9. $(^-3a)^2$ __________

10. $(n^3)^2$ __________

11. $(x^3)^3$ __________

12. $(2a^3)^2$ __________

13. $(^-4c^2)^2$ __________

14. $(a^2b^3)^2$ __________

15. $(^-5a^3)^2$ __________

Name ________________________________

Dividing Monomials

To simplify fractions with variables:

- Add or subtract exponents with like variables in the numerator and denominator.
- Simplify the coefficients.
- All exponents must be positive.

Example: $\dfrac{14x(x^5)y^2}{2x^4y^5}$

1. Simplify like variables in the numerator and in the denominator. $\dfrac{14x(x^5)y^2}{2x^4y^5} = \dfrac{14x^6y^2}{2x^4y^5}$
2. Simplify the coefficients. $= \dfrac{7x^6y^2}{x^4y^5}$
3. Simplify the x's and y's. $\left(\dfrac{x}{x} = 1,\ \dfrac{y}{y} = 1\right)$ $= \dfrac{7x^2}{y^3}$

Perform the divisions. Simplify, following the steps given.

1. $\dfrac{6x^6y}{x^3y^3}$ Remember $y = y^1$

2. $\dfrac{10a^3b}{2a^5b}$

3. $\dfrac{5(3x)x^2y}{3xy^3}$

4. $\dfrac{18an^5}{3a^3}$

5. $\dfrac{(3a)^2b^4}{3ab}$

6. $\dfrac{(^-2x)^3y^5}{(^-2y^2)^2}$

7. $\dfrac{(6ab^4)^2}{9ab^5}$

8. $\dfrac{6x^4y^4}{30(x^2y^2)^2}$

9. $\dfrac{20y^5z^6}{4(10y)z^3}$

10. $\dfrac{55m^5}{11m^3n}$

11. $\dfrac{^-3x^{10}y^8}{^-15x^{12}y^7}$

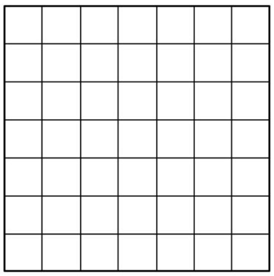
12. $\dfrac{(a^3b^2)^4}{(a^2b)^3}$

Name ________________________________

Square Roots of Monomials

The **square** of a number means to multiply a number by itself.
The square of 7 is 7 × 7 or 49.

The **square root** of a number is the opposite of the square of a number. The **radical sign** ($\sqrt{}$) indicates square root. $\sqrt{49}$ is 7 because 7 × 7 = 49.

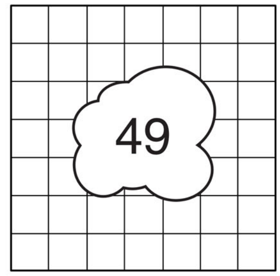


7

7

the square of 7 or "7 squared"

$7^2 = 7 \times 7$ or 49

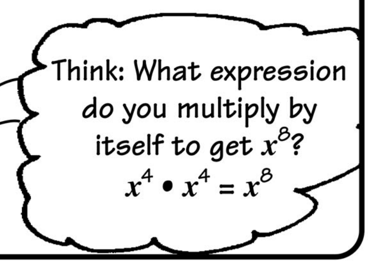


square root of 49 = 7

$\sqrt{49} = 7$

To find the square root of a monomial with a variable, break into numbers and letters. Find the square root of each part.

$$\sqrt{16x^8} = \sqrt{16}\ \sqrt{x^8}$$
$$= 4\ \ x^4$$



Find the positive square roots.

1. $\sqrt{81}$ **2.** $\sqrt{100}$ **3.** $\sqrt{121}$ **4.** $\sqrt{144}$

5. $\sqrt{r^{10}}$ **6.** $\sqrt{a^8}$ **7.** $\sqrt{16y^2}$ **8.** $\sqrt{49n^6}$

9. $\sqrt{100x^6}$ **10.** $\sqrt{81n^6}$ **11.** $\sqrt{9y^4}$ **12.** $\sqrt{4r^{10}}$

13. $\sqrt{4g^4h^6}$ **14.** $\sqrt{25x^4y^2}$ **15.** $\sqrt{64m^{10}}$ **16.** $\sqrt{16x^4y^2z^4}$

Name ________________________________

Patterns: Neither Arithmetic nor Geometric

A sequence is a set of numbers that follows a pattern.
Patterns may be arithmetic, geometric or neither.

Arithmetic	Geometric	Neither
3, 7, 11	5, 10, 20	1, 4, 9
+4 +4	×2 ×2	1^2 2^2 3^2
Add 4 repeatedly.	Multiply by 2 repeatedly.	Squaring consecutive natural numbers.

Find the next term in the pattern. Describe the rule.
Identify the pattern as arithmetic, geometric or neither.

1. 31, 38, 45, ______

Rule: ________________

Kind of pattern: ________________

2. 1, 4, 9, 12, ______

Rule: ________________

Kind of pattern: ________________

3. 5, 25, 125, ______

Rule: ________________

Kind of pattern: ________________

4. 1, 8, 27, ______

Rule: ________________

Kind of pattern: ________________

5. –4, –2, 0, ______

Rule: ________________

Kind of pattern: ________________

6. 81, 27, 9, ______

Rule: ________________

Kind of pattern: ________________

7. 3, 4, 6, 9, ______

Rule: ________________

Kind of pattern: ________________

8. 1, 4, 7, 10, ______

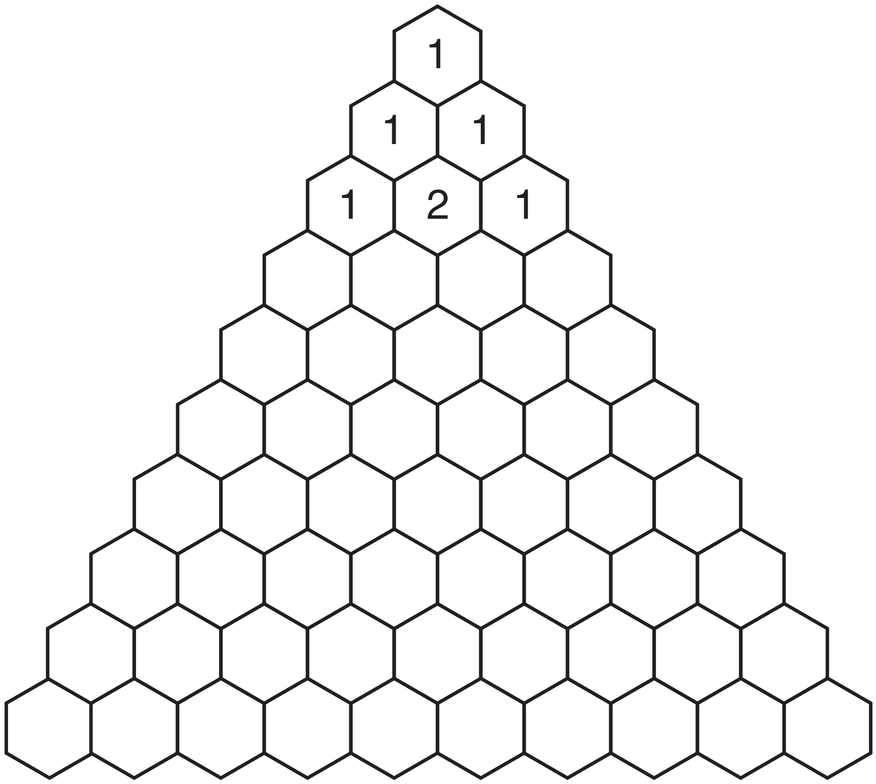
Rule: ________________

Kind of pattern: ________________

Name ___________________________________

Pascal’s Triangle

Pascal’s Triangle is a triangular array of numbers that includes many patterns.



Complete the numbers in Pascal’s triangle for the first 10 rows. Look for groupings of number patterns.

1. The _____ of any two adjacent numbers equals the number directly below and between the two numbers.

2. The sum of the numbers in each row is _____ times the total of the previous row.

3. The number in the cells along the outer sides of Pascal’s Triangle is always _____.

4. The numbers in the second diagonal are the set of _________ _________.

5. Another pattern I found was: ___________________________________.

Name ________________________________

Functions

A function is a set of ordered pairs that relate to each other by some rule. Functions can be represented by words, rules, tables or graphs.

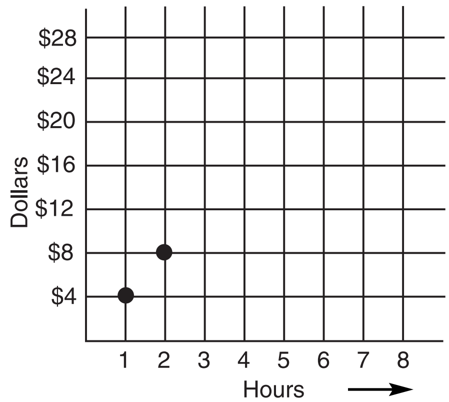
Shantel earns $4 an hour at her babysitting job. Complete a table and make a graph to show her earnings for 1 hour to 7 hours.

This earnings table is an example of a function.

hours	1	2	3	4	5	6	7
pay	$4	$8	__	__	__	__	__

Rule: multiply the number of hours worked by $4 to find the total earned.

Show the function on a coordinate grid to make a graph.

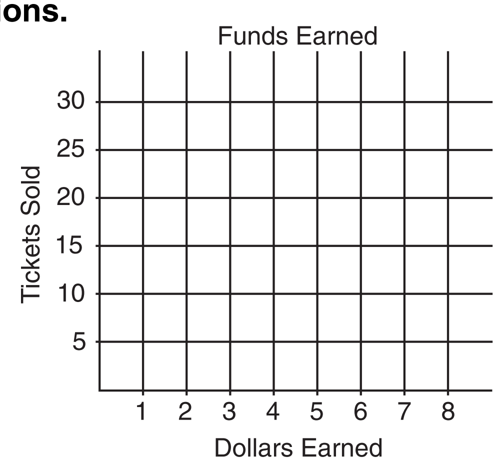


The first point is over 1 and up 4. The second point is over 2 and up 8. Complete the graph.

Make a table and graph to show the solutions.

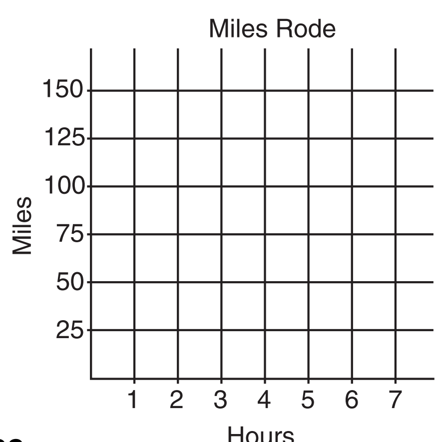
1. The Pep Squad earns $1.00 for every 5 raffle tickets they sell. Find the number of raffle tickets a club has to sell to make $2, $3, $4, $5.

earnings	$1	$2	$3	$4	$5
tickets	5				



2. Raul bikes at a rate of 25 miles per hour. How many miles does he bike in 2, 3, 4, 5 hours?

hours	1	2	3	4	5
miles	25				

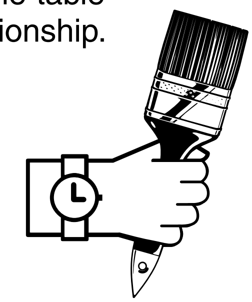


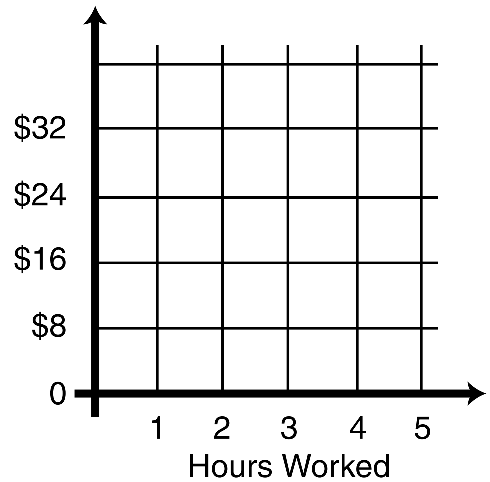
Name ___________________________________

Functions

Your summer job as a painter pays $8.00 an hour. Your pay depends on the number of hours you work. Your pay is a function of the number of hours you work. Complete the table and graph showing the relationship.

Hours	Pay
0	$
1	$
2	$



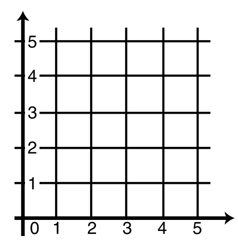


Connect the points.
What figure is shown? ___________

Make a table and complete a graph for 3 sets of ordered pairs for each function.

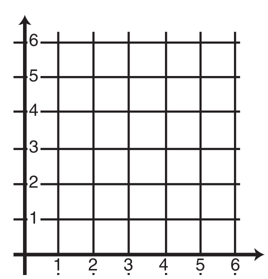
1. The sum of 2 numbers, *x* and *y*, is 5.

x	*y*
0	5
1	
2	



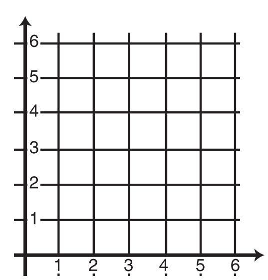
2. The difference between 2 numbers is 4.

x	*y*
5	1



3. The sum of 2 numbers is 6. Equation: ___________

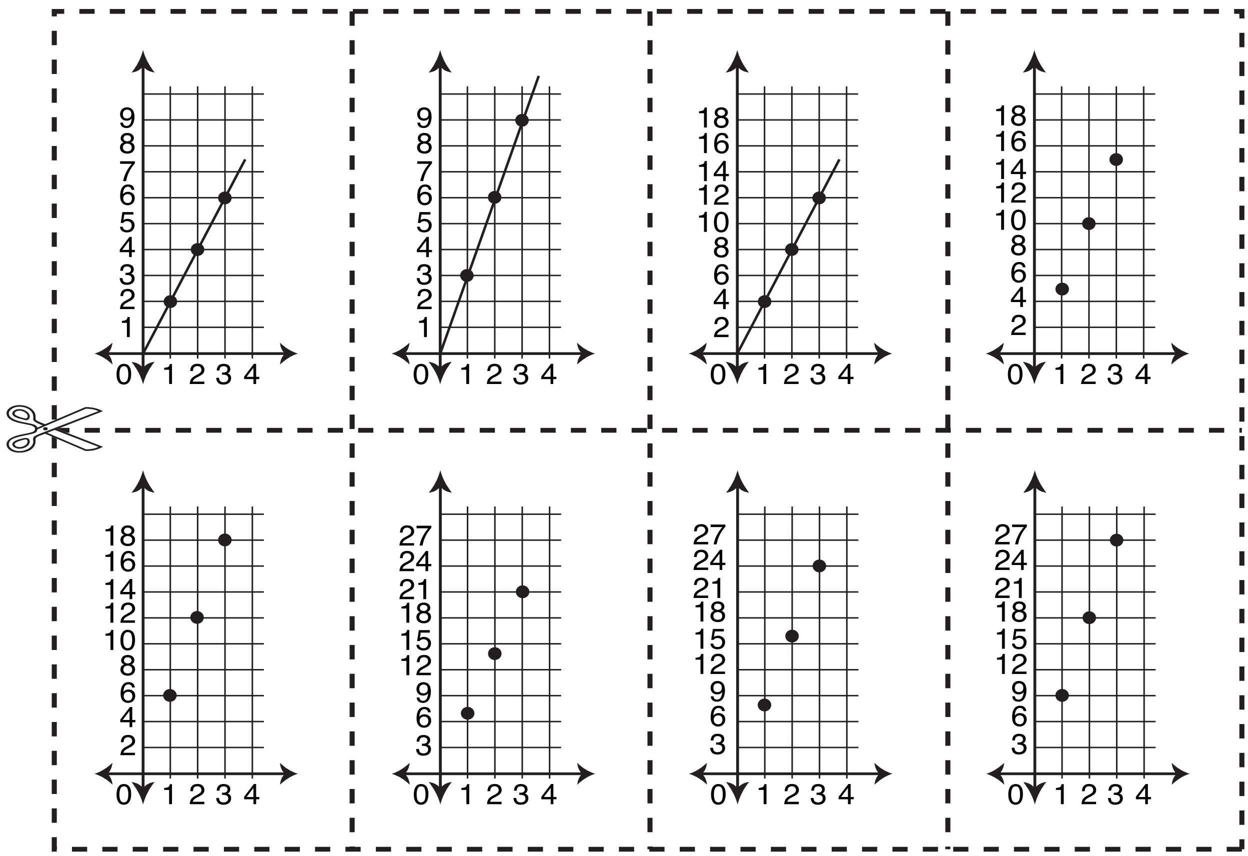
x	*y*
0	6



Representation Rummy (Page 1 of 2)

The object of the game is to form matching sets of a graph, a table and an equation. All 3 cards (graph, table and equation) in each set must represent the same mathematical fact.

1. Make copies of pages 238 and 239. Cut out the 8 cards on each page. Save these rules for player reference. Each group of 3 students should have 1 set (24 cards) made from the 2 pages.
2. The deck is shuffled and 4 cards are dealt to each of the 3 players. (The remaining cards will be used as a draw pile.) Players should check their cards to see if they have been dealt a matching set of 3 cards. If so, the set should be placed face up in front of that player and the player should take one replacement card from the draw pile.
3. The player to the left of the dealer begins play by asking either of his or her fellow players, “Do you have a different representation for this graph (or table or equation)?” If the player asked ***does*** have a matching representation, the matching card is surrendered to the first player. If the player asked ***does not*** have a match, the first player draws a card from the draw pile. Play passes (in a clockwise direction) to the player sitting to the left.
4. When a player completes a matching set (graph, table and equation) of 3 cards, the set should be placed face up in front of that player. That player then ***repeats*** his or her turn.
5. The hand ends when any of the players (after surrendering his or her last card or after laying down a set of 3 cards) has no cards left. Scores are totalled for that hand and each player receives 3 points for each representational set (of 3 cards) they have turned over.
6. The deck is reshuffled, and another hand is played. The first player to earn a total of 12 points wins the game.



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Representation Rummy (Page 2 of 2)

x	y
1	2
2	4
3	6

x	y
1	3
2	6
3	9

x	y
1	4
2	8
3	12

x	y
1	5
2	10
3	15

x	y
1	6
2	12
3	18

x	y
1	7
2	14
3	21

x	y
1	8
2	16
3	24

x	y
1	9
2	18
3	27

1 pint = 2 cups
$y = 2x$

1 yard = 3 feet
$y = 3x$

1 quart = 4 cups
$y = 4x$

1 hand = 5 fingers
$y = 5x$

1 hexagon = 6 sides
$y = 6x$

1 week = 7 days
$y = 7x$

1 box = 8 crayons
$y = 8x$

1 team = 9 players
$y = 9x$

Name ____________________________________

Points on a Line

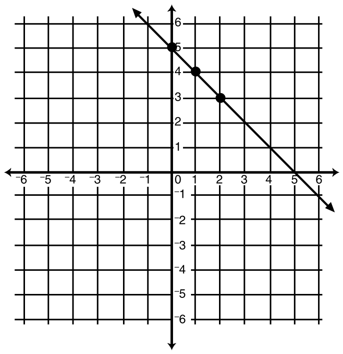
The degree of an equation is the same as that of its highest degree term.

first-degree equation: $x + 2y = 7$
second-degree equation: $x^2 + 4xy + 2y^2 = 0$
third-degree equation: $x^3 - x^2y + y^3 = 0$

A first-degree equation is called a **linear equation** since its graph is always a straight line.

Example: Graph the equation $x + y = 5$. Make a table and plot 3 points that make the equation true.

x	y
0	5
1	4
2	3



Does point (3, 2) lie on the line $x + y = 5$?

Does point (5, 1) lie on the line $x + y = 5$?

To check if a point lies on a line, substitute the value of the points into the equation.

Check: $x + y = 5$
$3 + 2 = 5$
$5 = 5$
Yes.

Check: $x + y = 5$
$5 + 1 = 6$
$6 \neq 5$
No.

For each equation, determine which points lie on the given line.

1. $3x + y = {}^-4$ ________

A $(1, {}^-7)$ **C** $(3, {}^-5)$
B $(2, 2)$ **D** $(0, {}^-4)$

2. $4x + 2y = 2$ ________

A $(0, 1)$ **C** $(2, 3)$
B $(1, {}^-1)$ **D** $(3, 5)$

3. $3x + y = 10$ ________

A $(1, 7)$ **C** $(3, 19)$
B $(2, 4)$ **D** $(0, 10)$

4. $y = x + 5$ ________

A $(1, 5)$ **C** $(3, 2)$
B $(2, 7)$ **D** $(0, 5)$

5. The point $(1, {}^-4)$ lies on which line? ________

A $y = 2x - 7$ **C** $y = \frac{x}{2} + 2$
B $y = x - 5$ **D** $y = 3x - 4$

6. The point $({}^-2, 1)$ lies on which line? ________

A $y = x - 1$ **C** $y = 2x + 5$
B $y = \frac{x}{2} + 1$ **D** $y = 2x - 3$

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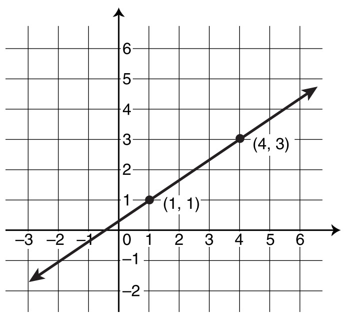
Name ____________________________________

Identifying the Slope of a Line

The **slope** tells you how steep a line is. The slope can be found by counting the number of units (up or down and then to the right) between any two points on a line.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{\# units up/down}}{\text{\# units to right}}$$

Find the slope of the line passing through (1, 1) and (4, 3)



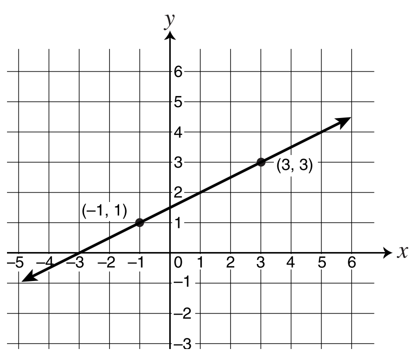
Start at the point farthest to the left (1, 1). How many spaces *up* until that point is even with the second point (4, 3)? ____

How many spaces *to the right* are there between the points? ____

$$\text{slope} = \frac{\text{\# units up/down}}{\text{\# units to right}} = \frac{2}{3}$$

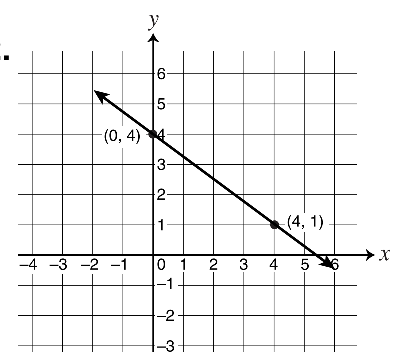
Given the two points on each graph, find the slope of each line.

1.



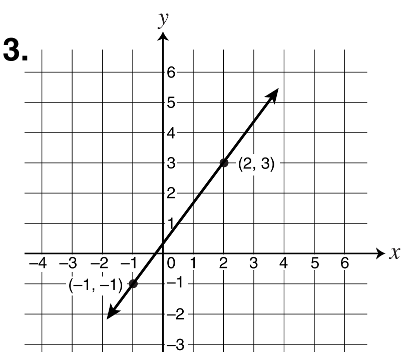
slope = ________

2.



slope = ________

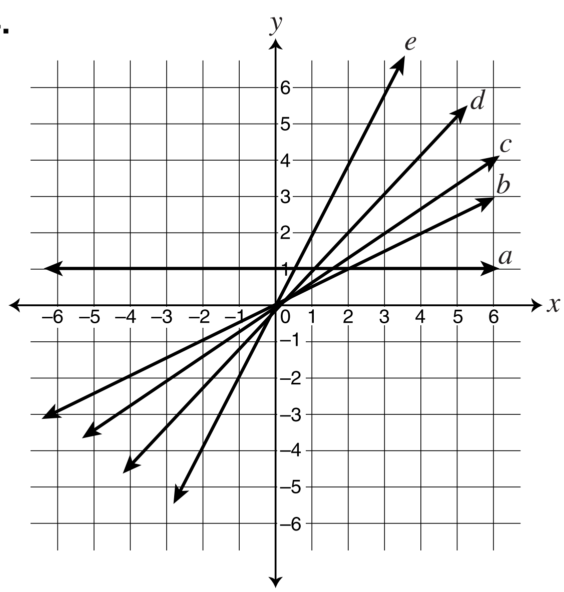
3.



slope = ________

Match the slopes with the lines on the graph.

4.



line *a*	slope = 2
line *b*	slope = $\frac{2}{3}$
line *c*	slope = 1
line *d*	slope = 0
line *e*	slope = $\frac{1}{2}$

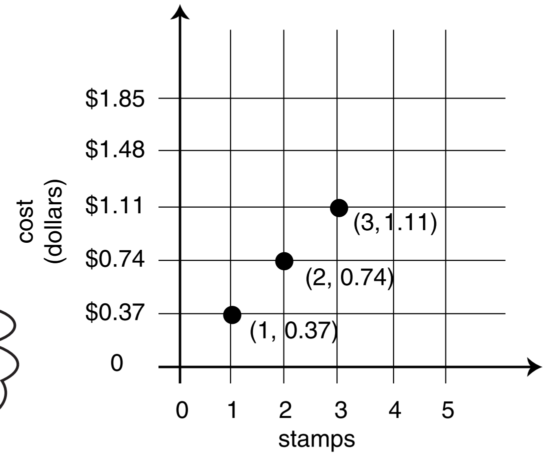
Name ______________________________________

Slope as a Ratio

Sometimes data is related by a constant ratio that can be expressed as a linear equation. To find the cost of $0.37 postage stamps, make a chart and graph the relationship between the number of stamps purchased and their cost:

number of stamps (x)	cost (y)
1	$0.37
2	0.74
3	1.11

Note: It is not appropriate to draw the line connecting the points. We can buy 3 stamps or 4 stamps but not 3.5 or 3.75 stamps.



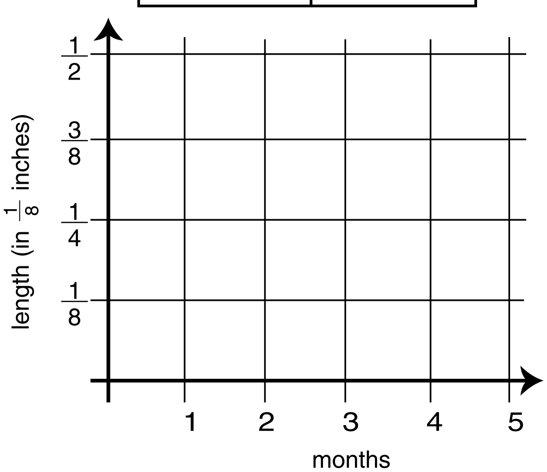
Scientists tell us fingernails grow $\frac{1}{8}$ inch per month and toenails grow $\frac{1}{16}$ inch per month. Complete the tables and graph the two lines representing these relationships.

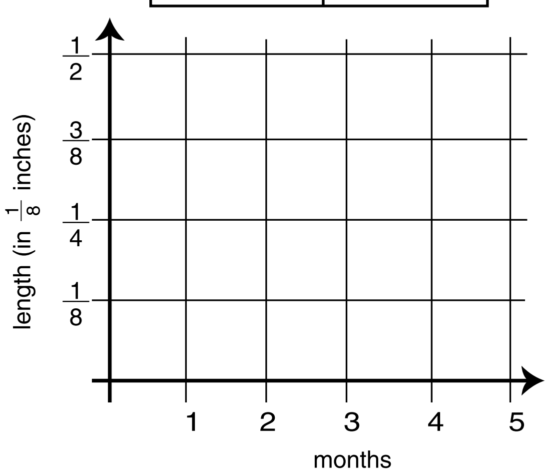
Fingernail Chart

month	length
1	$\frac{1}{8}$
2	
3	

Toenail Chart

month	length
1	$\frac{1}{16}$
2	
3	





1. How much will each grow in 4 months? Fingernail ________ Toenail ________

2. How much will each grow in 5 months? Fingernail ________ Toenail ________

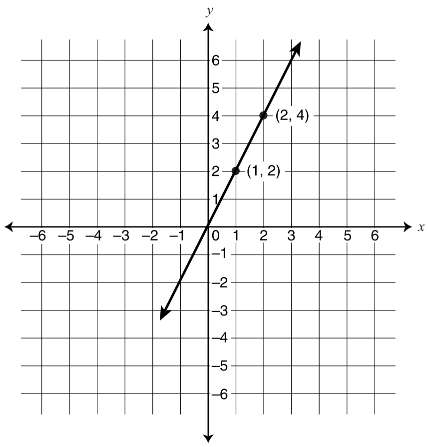
3. What is the slope of each graph? Fingernail ________ Toenail ________

Name ______________________________________

Using the Slope-Intercept Form

The slope-intercept form of an equation is $y = mx + b$ where m is the slope and b is the y-intercept.

The formula can be used to find the equation of a line from a graph.



Step 1: Find the slope.

$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-2}{2-1} = 2$$

Substitute the slope into the formula $y = mx + b$. $y = 2x + b$

Step 2: Substitute the coordinates of a point on the graph into the equation $y = 2x + b$.

Using point (1,2), $2 = 2 \bullet 1 + b$

$0 = b$

Thus, the equation of the line through points (1, 2) and (2, 4) is $y = 2x + 0$ or $y = 2x$.

The formula can also be used to determine if given points lie on the graph.

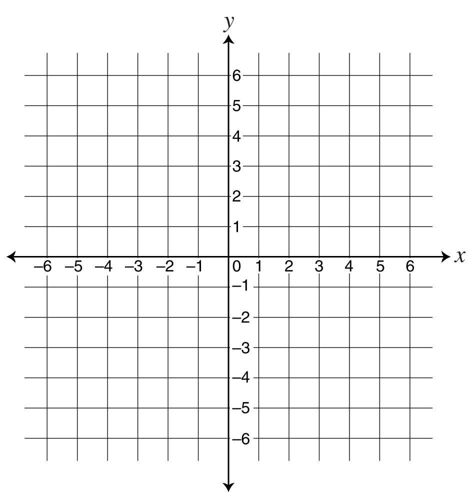
Example: Do (3, 1) and (4, $^{-}6$) lie on the graph of the line $3x + y = 6$?

Substitute (3,1) into the equation: $3 \bullet 3 + 1 = 6$, no.

Substitute (4,$^{-}6$) into the equation: $3 \bullet 4 - 6 = 6$, yes.

---

1. Graph points (1, 1) and (2, 3) on the coordinate grid. Then find the slope, y-intercept and equation for the line connecting the two points.



Slope = ________

y-intercept = ________

equation = _______________

2. The equation of a line is $y = {}^{-}3x + 6$. Do the points (3, 1) and (4, –6) lie on the line? (Hint: substitute each point into the equation.)

(3, 1) lies on the line. (T or F) _____

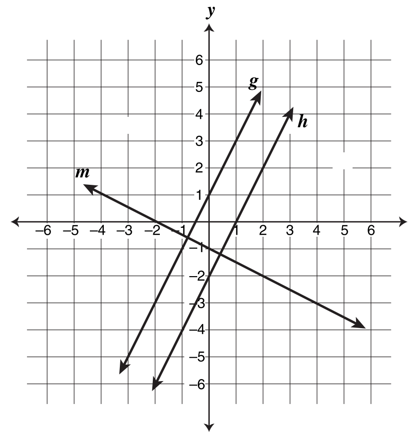
(4, $^{-}6$) lies on the line. (T or F) _____

3. Match the points to the line on which they lie.

$2x + y = 7$	(4, 4)
$y = 6x - 12$	($^{-}3$, 13)
$^{-}3y + 2x = {}^{-}4$	($^{-}3$, 12)
$y = -2x + 6$	(1, $^{-}6$)

Name ____________________________________

Parallel and Perpendicular Lines



Parallel lines never meet or intersect one another.
Perpendicular lines intersect each other at right angles.
The equations of lines g, h and m are written in slope-intercept form and are graphed on the grid.

line g: $y = 2x + 1$ line h: $y = 2x - 2$ line m: $y = {}^{-}\frac{1}{2}x - 1$

How are lines g and h related? __________

How are the slopes of g and h related? __________

How are lines g and m related? __________

How are the slopes of g and m related? __________

The product of the slope of two perpendicular lines = __________

Parallel lines have the same slope. The slopes of perpendicular lines have a product of $^{-}1$.

State whether the pair of lines is parallel, perpendicular or neither.

Are the slopes the same? Are the products of the slopes equal to –1?

1. $y = 6x + 7$
$y = 6x - 7$

2. $y = 2x + 4$
$y = {}^{-}2x + 4$

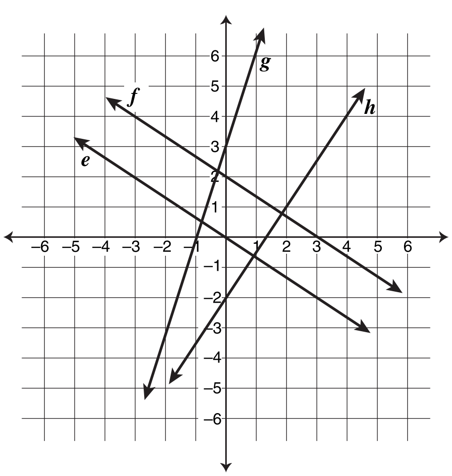
3. $y = \frac{1}{2}x$
$y = {}^{-}2x - 4$

4. $y = \frac{4}{3}x + 1$
$y = \frac{4}{3}x - 1$

5. $y = \frac{-2}{3}x + 6$
$y = \frac{3}{2}x - 4$

6. $y = {}^{-}4x + 2$
$y = \frac{1}{4}x - 3$

Use the graph below to answer problems 7 and 8.



7. Which lines are parallel to each other? __________

8. Name two pair of perpendicular lines. __________

9. Find the slope of line f. __________
What does this tell you about the slope of line e? __________

10. Find the slope of line h. __________

Name ____________________________________

Writing Equations for 2-Step Problems

Parentheses ( ) indicate that the operation inside the parentheses must be done before any other operations.
Example: Daniel got 5 boxes of football cards. Each box had 50 packs and each pack had 17 cards. Then he got a box that had 275 cards in it. How many cards did he get in all?

First, find the number of cards in 5 boxes.
Then, add 275 to the answer.

$(5 \times 50 \times 17) + 275 = n$

Use parentheses to write an open number sentence to solve each problem.

1. You won a lottery worth $900,000. If you are paid $10,000 a month, for how many years will you be paid?

____

2. A man walked from Seattle to Pittsburgh and back. It is 10,771,200 feet from Seattle to Pittsburgh. There are 5,280 feet in a mile. How many miles did he walk?

____

3. Viki likes to collect coins. She has 102 Mexican coins, 310 Spanish coins, 100 Egyptian coins, and 532 English coins. If her brother has twice as many as she does, how many coins does her brother have?

____

4. Charles was driving 5 miles to get to the store. When he got there he bought seven tubes of toothpaste. Each tube cost $2.76, but he had a coupon for a dollar off one tube of toothpaste. How much did all the toothpaste cost with the coupon?

____

5. Sara went to a sidewalk book sale at the mall. There were books by Hemmingway, Dickens, Fitzgerald, Shakespeare, and Auel. Sara had $20.00 to spend. She bought two books by Auel. They cost $2.50 each. How much did she have left?

____

6. Miguel sold hot dogs, potato chips, and soda. The hot dogs cost $1.45, the potato chips are $0.50, and the soda is $0.75. If twenty-five people bought hot dogs, forty-nine people bought potato chips, and seventeen people bought soda, how much did Miguel collect?

____

7. Tammy has started a business making jewelry and barrettes. She charges $4.00 for each pin, $3.00 for each barrette plus a $2.00 handling charge. Complete the tables to show the total charge for 1 to 10 barrettes and 1 to 10 pins.

No. of barrettes	1	2	3	4	5	6	7	8	9	10
Total Charge	$5.00	$8.00								

No. of pins	1	2	3	4	5	6	7	8	9	10
Total Charge	$6.00	$10.00								

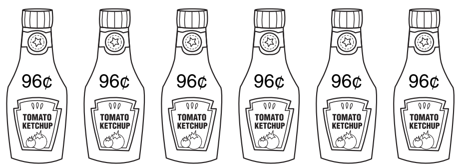
Name ____________________________________

Problem Solving: Multiple Price and Price Per Unit

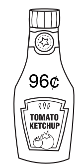
One helpful way to solve a word problem is to draw a picture of it. These two problems, which sound alike, become easier with a sketch.

Problem 1. A bottle of ketchup costs $0.96. How much will 6 bottles cost?

$$6 \times \$0.96 = \$5.76$$



Problem 2. A 6-ounce bottle of ketchup costs $0.96. What is the cost for one ounce (cost per ounce)?



$$\begin{array}{r} \$\ \ .16 \\ 6\,\overline{)\,\$0.96} \end{array}$$

Find the costs of the following:

1. 5 lb. of onion at $0.12 a pound ________

2. 6 chairs at $38.50 a chair ________

3. 24 cans of fruit at $0.59 a can ________

4. Annual gas cost at $75.40 a month ________

Find the cost per unit of the following:

5. A 5 lb. bag of flour for $1.65 ________

6. A 4 oz. package of cheese for $1.96 ________

7. An 8 oz. jar of peanut butter for $1.36 ________

8. A 6 oz. can of juice for $0.78 ________

Solve.

9. Oranges cost $0.27 per pound. How much will 10 pounds cost? ________

10. A fuel oil consumer bought 100 gallons of heating oil at $0.495 per gallon. How much did he pay for the oil? ________

11. If four boxes of candy cost $7.56, how much will 1 box cost? ________

12. If 3 kilograms of cheese cost $7.62, how much does 1 kg cost? ________

13. How much hamburger is needed to make 15 hamburgers if each patty takes 0.52 pound of meat? ________

14. A package of 6 pencils costs $0.72. How much does one pencil cost? ________

Name ___________________________________

Powers of Monomials

n^3 exponent
base

n is called the base.
3 is called the exponent.
n^3 means multiply n by itself three times
$n^3 = n \bullet n \bullet n$

$(n^3)^2$ means to square n^3 or to multiply n^3 by itself.

$(n^3)^2 = n^3 \bullet n^3$
$= n \bullet n \bullet n \bullet n \bullet n \bullet n$
$= n^6$

Write the following expressions with exponents.

1. $x \bullet x \bullet x$ **2.** $n \bullet n$ **3.** $2 \bullet x \bullet x$ **4.** $3 \bullet y \bullet y \bullet y$

5. $4 \bullet n \bullet n$ **6.** $a \bullet a$ **7.** $3 \bullet x \bullet x$ **8.** $2 \bullet y \bullet y$

Write the following expressions in factored form.

9. c^2 **10.** n^4 **11.** a^2b **12.** y^3

13. m^2 **14.** x^3 **15.** m^2n^2 **16.** $3y^2$

Find the indicated squares.

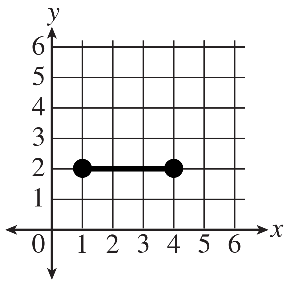
17. 3^2 **18.** $(xy)^2$ **19.** 9^2 **20.** 10^2

21. $(x^3)^2$ **22.** $(y^4)^2$ **23.** $(ab)^2$ **24.** $(xy^2)^2$

Name ______________________________________

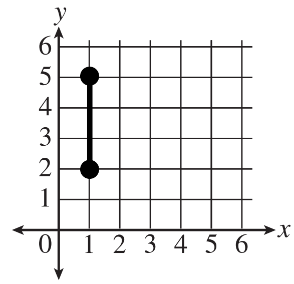
Horizontal and Vertical Line Segments

You can find the length of a line segment in a coordinate grid by subtracting.



The length of the horizontal line equals the difference in the *x*-coordinates.

$x_2 - x_1 = 4 - 1 = 3$

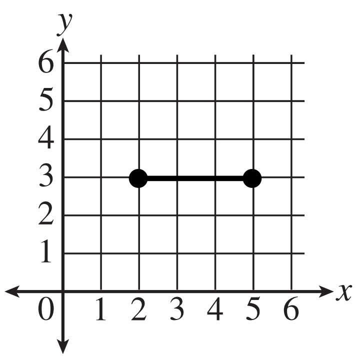


The length of the vertical line equals the difference in the *y*-coordinates.

$y_2 - y_1 = 5 - 2 = 3$

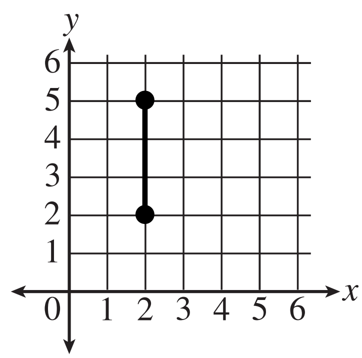
Find the length of each line segment.

1.



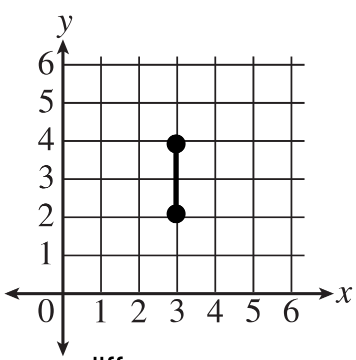
$x_2 - x_1 =$

2.



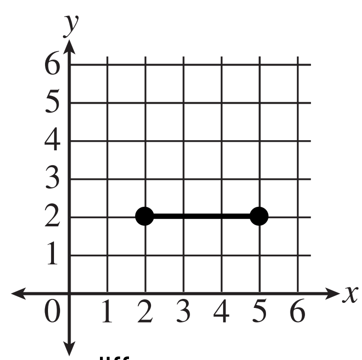
$y_2 - y_1 =$

3.



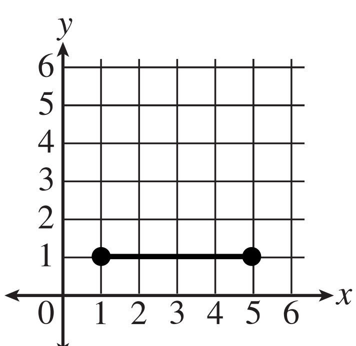
difference =

4.



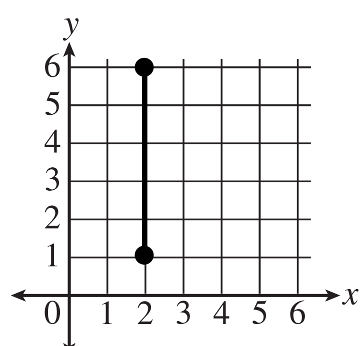
difference =

5.



difference =

6.



difference =

Name ______________________________________

Using the Slope-Intercept Form in Graphing

The graph of a line $y = mx + b$ has the slope m and the y-intercept at $y = b$. When a linear equation is written in the form of $y = mx + b$, we know the slope and y-intercept and can draw the graph.

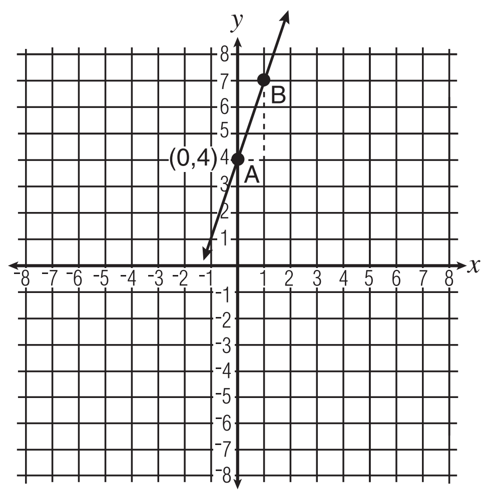
Graph $y = 3x + 4$

Step 1. The y-intercept is 4.
Plot Point A at (0,4)

Step 2. The slope is $^{+}3$ or $\frac{3}{1}$.

An increase of 1 in x produces an increase of 3 in y. Start at Point A and go 1 unit to the right and 3 units up to find a second point, B.

Step 3. Draw a line through points A and B.



Give the slopes and y-intercept of the following equations.

1. $y = 3x + 1$
slope: __________
y-intercept: _________

2. $y = 2x + 5$
slope: __________
y-intercept: _________

3. $y = \frac{1}{2}x$
slope: __________
y-intercept: _________

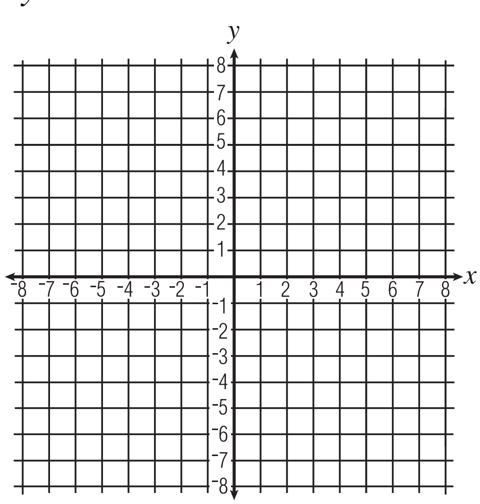
4. $y = {}^{-}2x + 3$
slope: __________
y-intercept: _________

5. $y = \frac{-3}{4}x + 1$
slope: __________
y-intercept: _________

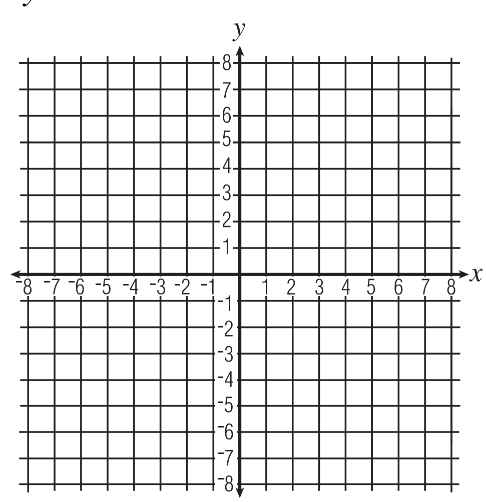
6. $y = \frac{1}{2}x + 3$
slope: __________
y-intercept: _________

Graph the equations by using the y-intercept and slopes.

7. $y = 3x + 2$



8. $y = 2x - 1$



Name ______________________________

Solving Equation Puzzles

Cut out Problem A and tape it on the outside of a business envelope. Cut out the A equations and justifications (cut out each equation and justification separately) and put them into the envelope. Do similarly for Problems B, C, and D. Give one envelope to each group. Have students arrange the equations in proper order and line up the justifications next to the appropriate steps.

Problem A. Solve for ***x***. $5x + 6 = 31$

$5x + 6 = 31$ A	Given A
$5x + 6 - 6 = 31 - 6$ A	Addition/Subtraction property of equality A
$5x + 0 = 25$ $5x = 25$ A	Combining like terms A
$\frac{5x}{5} = \frac{25}{5}$ A	Multiplication/Division property of equality A
$\frac{5}{5} \bullet \frac{x}{1} = \frac{25}{5}$ $x = 5$ A	Simplifying fractions property of equality A

Problem B. Solve for ***x***. $4x + 9 = 16 - 3x$

$4x + 9 = 16 - 3x$ B	Given B
$4x + 9 - 9 = 16 - 3x - 9$ B	Addition/Subtraction property of equality B
$4x + 0 = 7 - 3x$ $4x = 7 - 3x$ B	Combining like terms B
$4x + 3x = 7 - 3x + 3x$ B	Addition/Subtraction property of equality B
$7x = 7 - 0$ $7x = 7$ B	Combining like terms B
$\frac{7x}{7} = \frac{7}{7}$ B	Multiplication/Division property of equality B
$\frac{7}{7} \bullet \frac{x}{1} = \frac{7}{7}$ $x = 1$ B	Simplifying fractions B

Name ________________________________

Solving Equation Puzzles

Problem C. Solve for *x*. $3(x + 6) = 6x$

$3(x + 6) = 6x$ C	Given C
$3x + 18 = 6x$ C	Distributive property C
$3x - 3x + 18 = 6x - 3x$ C	Addition/Subtraction property of equality C
$0 + 18 = 3x$ C	Combining like terms C
$\frac{18}{3} = \frac{3x}{3}$ C	Multiplication/Division property of equality C
$\frac{18}{3} = x \quad x = 6$ C	Simplifying fractions C

Problem D. Solve for *x*. $5(3x - 20) = -10$

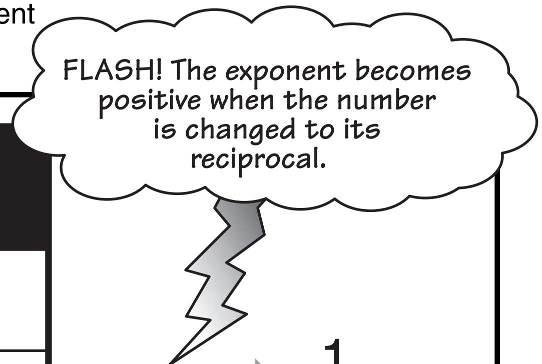
$5(3x - 20) = -10$ D	Given D
$15x - 100 = -10$ D	Distributive property D
$15x - 100 + 100 = -10 + 100$ D	Addition/Subtraction property of equality D
$15x + 0 = 90$ D	Combining like terms D
$\frac{15x}{15} = \frac{90}{15}$ D	Multiplication/Division property of equality D
$\frac{15}{15} \bullet \frac{x}{1} = \frac{90}{15} \quad x = 6$ D	Simplifying fractions D

Name ______________________________

Negative Exponents

To write a number having a negative exponent as a number having a positive exponent, write the reciprocal of the number and change the exponent from a negative to a positive.

Number with negative exponent	Number with positive exponent	Expanded form
3^{-1}	$\frac{1}{3^1}$	$\frac{1}{3}$
3^{-2}	$\frac{1}{3^2}$	$\frac{1}{3 \bullet 3}$
3^{-3}	$\frac{1}{3^{\square}}$	$\frac{1}{\_ \bullet \_ \bullet \_}$
3^{-4}	$\frac{1}{3^{\square}}$	$\frac{1}{\_\_\_}$
3^{-5}		
3^{-6}		



$$3^{-2} \rightarrow \frac{1}{3^2}$$

Write each expression with a positive exponent, then evaluate.

1. $5^{-4} = \frac{1}{5^4}$

$= \frac{1}{5 \bullet 5 \bullet 5 \bullet 5}$

2. 3^{-2}

3. 7^{-3}

4. 6^{-2}

5. 4^{-3}

6. 3^{-5}

7. 2^{-3}

8. 5^{-3}

Write with a positive exponent.

9. y^{-5}

10. x^{-3}

11. z^{-4}

12. x^{-6}

13. n^{-2}

14. m^{-1}

15. $\left(\frac{1}{3}\right)^{-3}$

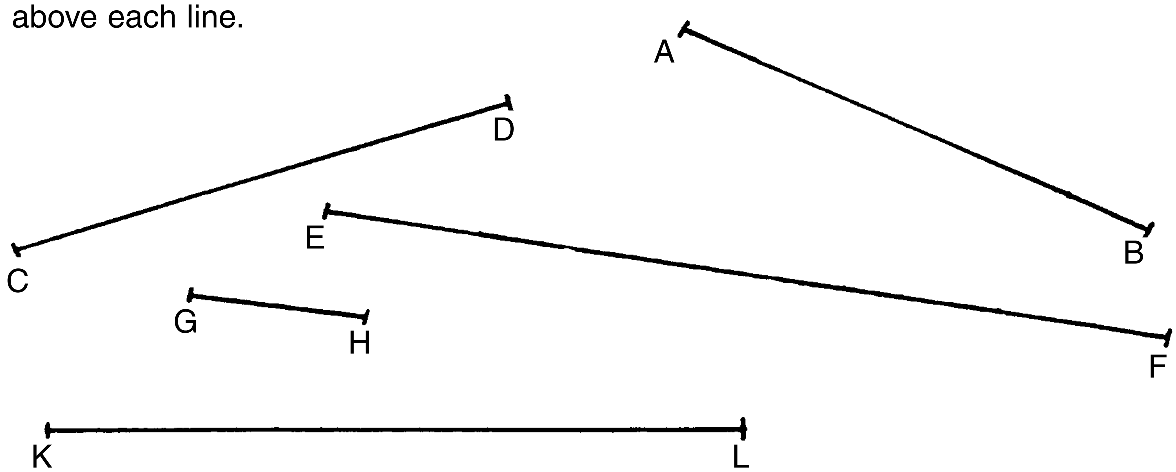
16. $\left(\frac{3}{4}\right)^{-2}$

Name ________________________________

Making and Using a Ruler

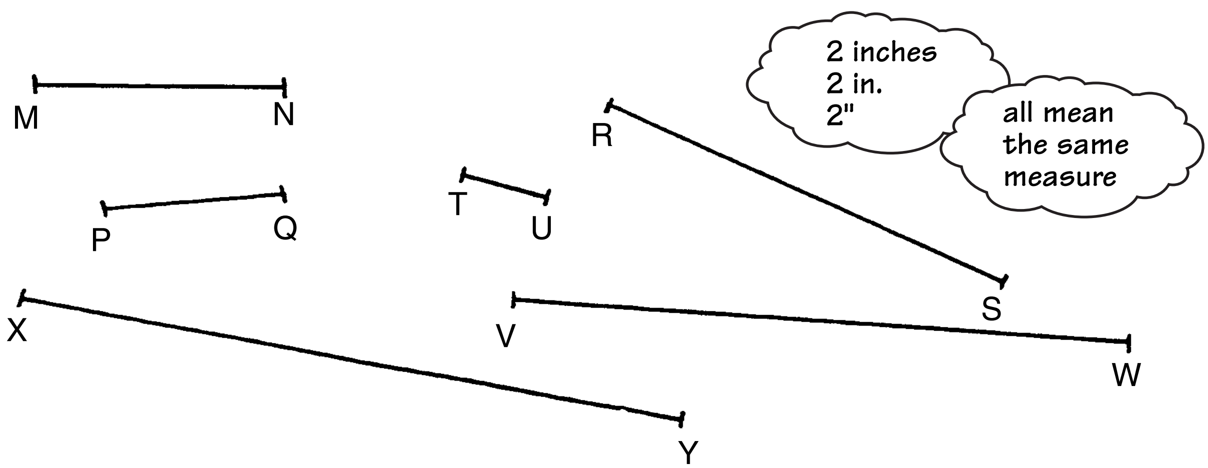
1. Cut out the 1-inch measure. Use it to mark off 1-inch lengths along the edge of the ruler. Label each mark 0, 1, 2, 3, and so on.

2. Cut out your ruler. Measure the lines below to the nearest inch. Write the measure above each line.



3. Cut out the $\frac{1}{2}$ inch measure. Use it to mark off $\frac{1}{2}$ inch lengths along the edge of the ruler. Label each mark 0, $\frac{1}{2}$, 1, and so on.

4. Cut out your ruler. Measure the lines below to the nearest $\frac{1}{2}$ inch. Write the measure above each line.

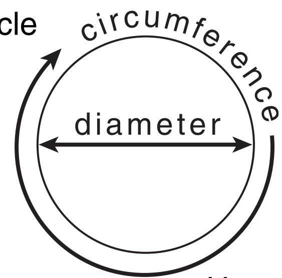


Relating Ordered Pairs of Functions to Slope

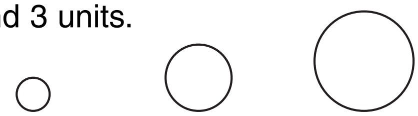
The ratio of the vertical change (change in *y*-value) to the horizontal change (change in *x*-value) in a functional relationship is called the slope of a graph.

The circumference of a circle is approximately equal to 3 times the diameter.

$$C \cong 3 \bullet d$$



The table shows the relationship for circles with diameters of 1 unit, 2 units, and 3 units.



d = 1 unit *d* = 2 units *d* = 3 units

diameter (*x*)	circumference (*y*)
1	3
2	6
3	9

The ratio of the circumference of each circle to its diameter is:

d of 1 = $\frac{3}{1}$

d of 2 = $\frac{6}{2}$ or ______

d of 3 = $\frac{9}{3}$ or ______

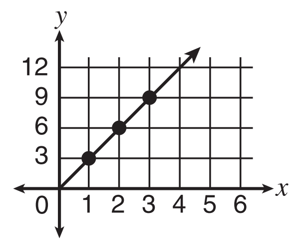
How do the ratios compare to each other?

____

This ratio is called the **slope** of the line.

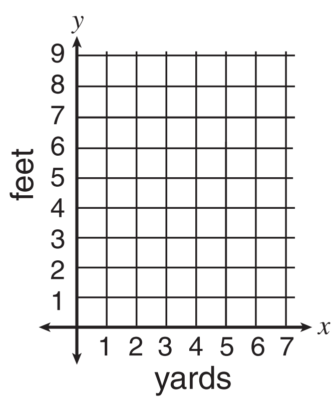
Slope of the line: _____

We can graph the slope of the line:



1. The ratio of feet to yards is shown by the expression: number of feet = 3 times the number of yards. **Complete the table for the relationship. Graph the relationship. Find the slope of the line.**

Number of yards (*x*)	Number of feet (*y*)	Ratio of *y* to *x*
1		
2		
3		

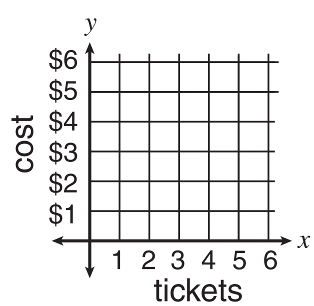


slope of the line:

____

2. A ticket for a ride at the amusment park is $2. **Complete the table showing the cost of 1, 2, and 3 tickets. Graph the relationship. Find the slope of the line.**

Number of tickets (*x*)	Cost of tickets (*y*)	Ratio of *y* to *x*
1	$	
2	$	
3	$	



slope of the line:

____

MA-39, MA-40